=$;NO@J8

JBNs2=% 3% s% Q% $% iTINPAR K *$ 1$ k <+F 0% G! <% ?J ,3 d7 hD j< jK !

;XF 36541 IYED bC<# 65<x

5~ETBg3XBg3X1 !9)3X8 &5 f2J
=$;N2]Dx>pJs9)3X@196

C$L& > 08c

J?@.10 G/2 7n13 F |

JBNs2=%3%s%Q%$%i **TINPAR** K*1k<+F0%G!<%?J,3d7hDj<jK!

C$L&>08c

FbMF9<35

6aG/!"9b@-G=7W;;5!H7$F!"JBNs7W;;5!$KBg$-$J4|BT$,4s$;i1$F$$$k!#7+$7!"01HL$K!"JBNs7W;;5!8~$-$N%W%m%0%i%`$N5-=R$O:$Fq$G$"$k$?$a!"C`<!7W;;5!8~$-$K5-=R$5$1$?%W%m%0%i%`$r<+F0E*$KJBNs2=$9$k!"<+F0JBNs2=%3%s%Q%$%i$N<B8=$,5aai1F$$$k!#2f!9$O8=:_!"$3$&$$$C$?MW5a$r<B8=9Y$/!"%a%C%;!<%88r497?JBNs7W;;5!8~1NJBNs2=%3%s%Q%$%i **TINPAR** $r3+H/Cf$G$"$k!#

3Nh&$J%a%C%;!<%88r497?JBNs7W;;5!8~$1JBNs2=%3%s%Q%$%i$O!"%f!<%6$KhC$F;XDj$5$1$?%G!<%?J,3d$K4p$E$-!"Owner Computes Rule $K=>$$$5$^$6$^$JJBNs2=!":GE,2=$r9T$&bN$,B?$$!#$3$1iNJBNs2=%3%s%Q%$%i$N<B8=Kh$j!"%f!<%6$O%W%m%0%i%`$N=hM}$NJ,3d$dDL?.%3!<%I$NA^F~$H$$$C$?HQ$o$7$$:n6H$+$i2rJ|$5$1$k!#$7$+$7!"0MA3$H$7$F%G!<%?J,3d$N;XDj$O%f!<%6$N@UL3$H$5$1$?$^$^$G$"$k!#%f!<%6$,:GE,$J%G!<%?J,3d$r5a$a$k$?$a$K$O!"$=$N%G!<%?J,3d$K$h$j!"%W%m%0%i%`$,$INh$&$KJBNs<B9T$5$1k+$r2r@O$9kI,MW,$"$k$?$a!"01HIE*KOHs>o$K:$FqJ3HG$"$k!#

$=$3$GK\8&5f$G$O!"JBNs2=%3%s%Q%$%i$KM?$($k%G!<%?J,3d$r!"C`<!%W%m%0%i%`$+$i<+F0E*$K7hDj$9k?aN<jK!$rDs0F$9$k!#

K\<jK!GO!"JBNs<B9T;~$N@-G=$KBg$-$J1F6A$rM?$($k9`L\$H7F%W%m%;%C%54V$NDL?.NL!"Cj=P$5$1$kJBNs@-!"%W%m%;%C%54V$NIi2Y%P%i%s%9$N3DKCeL\$7!"$3$1ir:GE,2=9k%G!<%?J,3d$r5a$ak3HrL\E*H9$k!#

%G!<%?J,3d7hDj$O!"(1) G[Ns<!85%"%i%$%a%s%H$N7hDj!"(2) %V%m%C%/%5%$%:Hf$N7hDj!"(3) %V%m%C%/%5%$%:$N7hDj!"(4) J,3d<!85$N7hDj$N4$D$N%9%F%C%W$Khj9=@.$5$1$k!#

G[Ns<!85%"%i%$%a%s%H$N7hDjGO!"0[JkG[NsJQ?t4V$N<!85$NBP1~E1$r5a$a$k!#%V%m%C%/%5%$%:Hf$N7hDj$G$O!"G[Ns<!85%"%i%$%a%s%H$N7hDj$KhC$FBP1~$E$1$5$1$?<!854V$NJ,3d$N%V%m%C%/%5%$%:$N4X78$r5a$a$k!#$3$1$i$N3F%9%F%C%W$G$O!"C`<!%=!<%9%W%m%0%i%`$NG[NsJQ?tI&7NBeF~J8$N2r@O7k2L$rbH$K!"DL?.NL$r:GE,2=9kh&$K5a$ai1$k!#

%V%m%C%/%5%$%:$N7hDjGO!"A0%9%F%C%W$N%V%m%C%/%5%$%:Hf$NHO0OFb$GHjF@$kJ,3d$N%V%m%C%/%5%$%:$NCf$+$i!"DL?.NL!"Ii2Y%P%i%s%9$r:GE,2=$9kh$&

$J%V%m%C%/%5%$%:$r5a$a$k!#

J,3d<!85$N7hDj$G$O!"$3l^GN%9%F%C%W$G5a$^C?G[NsJQ?t$NJ,3d$K4X$9 $k7hDj;v9`$rK~$?$9HO0OFb$G!"J,3d<!85$N8uJd$r@8@.$7!"@8@.5l$?3F8uJd$KBP 7F@-G=8+@Q$r9T$&3H$G:GE,$JJ,3d<!85$r7hDj$9kH$$$&%"%W%m!<%ArH$k!#

$3$3$G;HMQ$5lk@-G=8+@Q<jK!$O!"C`<!%=!<%9%W%m%0%i%`$N%G!<%?0MB82r@O >pJsrbHK!"M?$($il?%G!<%?J,3dKBP9$kJBNs<B9T;~4V$N8+@Qjr9T$&!# K\<jK!$G$O!"$^$:%G!<%?0MB84X78$N$?$a$KH/@8$9$k%W%m%;%C%54V$G$N7W;;$N<B 9T=g=x$N@)Ls$r5aak3HKh$j!"DL?.;~4V$r0 H7$?>l9g$NJBNs<B9T;~4V$r 8+@Q$k!#B3$$$F!"DL?.$,H/@8$9$k;~E@$H!"$=$l$>$l$NDL?.$NDL?.Aj<j!"DL?.NL $r5a$a$k!#:G8e$K>e5-$N3F;~4V$r9g7W7F!":G=*E*$JJBNs<B9T;~4V$r8+@Q$k!#

K\8+@Q<jK!GO!"ESCf$GDL?.$r9To:$K<B9T$r9T$($k=hM}N&$A!"2DG=$J8B jBg-$J=hM}C10L$r4pK\%?%9%/=89g$H8F$V!#4pK\%?%9%/=89gFb$N=hM}$O9Y$F F10l%W%m%;%C%5$G<B9T5l!"Aw<u?.$O!"4pK\%?%9%/=89g$N:G=i$H:G8e$GN_9T ol$k!#$3$N35G0$NF3F~Kh$j!"DL?.$H7W;;$N=g=x$d!"$=$l$>$lNNLN8+@Q$j $r@53N$K9T$&$3H,2DG=HJC?!#

JBNs2=%3%s%Q%$%iTINPAR KhCF=PNO5l$k%3!<%I$r%a%C%;!<%88r4 97?JB Ns7W;;5!AP1000 $G<B9T$9k3HrA[Dj$7!"livermore kernel No.23 $N%W%m%0%i % `$KBP$7$FK\%G!<%?J,3d7hDj<jK!$rE,MQ$7$F!"3FG[NsJQ?t$NJ,3d$r7hDj$7$?!# $5$^$6$^$J%k!<%W$N<B9T2s?t!"%W%m%;%C%5Bf?t$KBP$7$F8+@Q;~4V$r;;=P$7$?7k 2L!"%W%m%;%C%5Bf?t!"%k!<%W$NH?I|2s?t$K$h$C$F:GE,$H$J$kJ,3d$,0[$J$k$H$$$& 7k2L$,F@$i$l$?!#<B:]$KAP1000 $G%W%m%0%i%`$r<B9T7FF@il$?7k2L$HHf3S $7!":GE,$J%G!<%?J,3d$,0[$J$k>r7o$r$+$J$j@53N$K5aail?3H$r3NG'$7$?!#

An Automatic Data Distribution Method for Parallelizing Compiler TINPAR

Shougo TATSUMI

Abstract

Parallel machines are regarded as the next generation supercomputers. However, writing well-tuned programs is not always easy. For this reason, there are requirements for parallelizing compilers which automatically generate parallelized code for message-passing multiprocessors from the given sequential programs without complicated programming. Therefore, we are now developing TINPAR: a parallelizing compiler for message-passing multiprocessors.

This type of compilers parallelize sequential code with Owner Computes Rule and perform many optimizations based on the specified data distribution pattern. Thus, users can use multiprocessors without complicated parallel programming. However, users still have to specify the data distribution pattern. The determination of the optimal data distribution pattern is very difficult for users to specify because it requires a very complicated analysis parallel execution to predict performance.

To release the users from this burden, this paper presents an automatic data distribution method for a parallelizing compiler for message-passing multiprocessors.

The performance of programs running on multiprocessors depends on the overheads communication, the exploited parallelism. the load balance between processors. Hence, our policy of determination of data distribution pattern is to optimize these three issues.

Our automatic data distribution method consists of the following four steps: (1)decision of the alignment of arrays dimensions, (2)decision of the block-size constraints, (3)decision of block-sizes of the arrays dimensions and (4)decision of array dimensions to be distributed.

First two steps analyze the relations about the distribution between all array variables. Our automatic data distribution method obtains these relations by analyzing assignment statements containing arrays in both sides to minimize the volume of communications.

In the next step, the blocksize, which optimizes the volume of communication and the load balance, and which satisfies the blocksize constraints determined in the previous step, is selected.

To determine array dimensions to be distributed, our method generates all possible candidates of data distribution patterns so that the relations obtained in the previous steps are kept, and choose one data distribution pattern with which the parallelized program is estimated that the parallel execution time is minimum.

The performance estimator used in this step estimates the parallel execution time using data dependence information for the given programs with the specified data distribution pattern. For the first step, we estimate the execution time of the parallel program by obtaining the ordering constraints between processors assuming that all communications take no latency. Then, we estimate when the communications occur, the recipient of the data elements and the volume of data. For the last step, we sum up the overall execution time and then we obtain the parallel execution time.

We use the concept of the *Basic Task Set* – the set of computations which is executed by the same processor with no communications – to estimate the order of computations and communications and the volume of each communication. The estimated time becomes accurate by introducing the concept of the Basic Task Set to our estimation scheme.

We decide the data distribution pattern of livermore kernel No.23 using our automatic data distribution scheme assuming that it will be parallelized by TINPAR and will be executed on message-passing multicomputer AP1000. As the result of the estimations of the parallel execution time for this program with some combinations of the value of the parameters such as the number of iterations or the number of processors, the optimal data distribution pattern changes according to the combinations of the value of these parameters. As the result of the comparison between the estimated time and the measured time, we confirm that we can choose the optimal data distribution pattern for given combinations of the value of these parameters for this program.

JBNs2=%3%s%Q%$%i **TINPAR** K*1k<+F0%G!<%?J,3d7hDj<jK!

L\<!

1 O8aK

9b@-G=7W; ;5 !H7$F! "JBNs7W; ;5 !$KBg$-$J4 |BT$,4s$;il$F$$$k! #FC$K%a%C %;!<%88r4 97 ?$NJBNs7W; ;5 !$ODL? .%3!<%I$rM [$K5-=R$9k3HG%W%m%0%i% `$r9b EY$K:GE,2=$G$-$k$N$G! "9b$$<B8z@-G=$rC#@ .$9$k$3$H$,4 |BT$G$-$k! #$7$+$7<B :]$K$O! "9b$$<B8z@-G=$rC#@ .$G$-$k%W%m%0%i% `$r:n@ .$9kK$O! "9bEY$J5 ;=Q$d CN<1$,I,MWG"$j! "$3li$,JBNs7W; ;5 !$NIa5Z$rAK32$9$k0 l0x$HJC$F$$$k! #

3&$$$C$?:$Fq5r9nI~9k$?$a$K! "$$$+$K7W; ;$r5;k+$H$$$&%W%m%0%i %_%s%0$NK\<AE*$J3HK@$1=8Cf$7$FC `<!%W%m%0%i% `$r5-=R$7! "$=lrJBNs7W ; ;5 !MQ$KJQ4 9$9$kJBNs2=%3%s%Q%$%i$N<B8=$,K>$^$l$F$$$k! #2f! 9$O8=:_! "$3$l iNMW5a$r<B8=$9Y/! "%a%C%;!<%88r4 97 ?JBNs7W; ;5 !8~1NJBNs2=%3%s%Q%$%i **TINPAR**[1] $r3+H/Cf$G$"$k! #

JBNs7W; ;5 !$G8zN ($h$/LdB j$r2 r$/$K$O! "%W%m%;%C%54V$NDL? .NL$rDc8 :$9k3 Hd! "LdB j$N; }$DJBNs@-$r:GBg8BC j=P$9k3H,=EMWG"$k! #=>Mh$h$j! "%G!< %?J,3d$,M?$ (ilkH! "3l$r4p$K:GE,@-G=$r=P$9JBNs2=%3%s%Q%$%i$,?tB ?$/ Ds0F$5lF$$$k! #7+$7! "JBNs2=%3%s%Q%$%i$KM?$ (il$k%G!<%?J,3d$O3l$i $N9 `L\$r:81 &$9$k=EMW$J%Q%i%a!<%?$G$"$k$3$H$K2C$ (! "$3$l$i$N3F9 `L\$r:GE, K9kh$&$J%G!<%?J,3d$N7hD j$O! "JBNs<B9T; ~$N%W%m%;%C%5$N?6I q$$$r9MN8$9 kI,MW,$"$kNG0 lHIE*KO:FqJ3HG"$k! #

7?$,$C$F! "$3li$,<+F02=$5ll$P! "4 0A4$J<+F0JBNs2=%3%s%Q%$%i$,<B8= G-! "%f!<%6$NI iC4$OBgI }$K7Z8 :$5lk! #

$=$3$GK\O@J8$G$O! "JBNs2=%3%s%Q%$%iK*$$$F! ":GE,$J%G!<%?J,3d$rC `<!%W %m%0%i% `$+$i<+F0E*$K7hD j$9$k<jK !$rDs0F9k! #K\<jK !GO! "(1) %W%m%;%C%54V $NDL? .NL! "(2) C j=P$5lkJBNs@-! "(3) %W%m%;%C%54V$NI i2Y%P%i%s%9! "$r:GE,2= 9k3H$rL\E*$H7?>e$G! "G [Ns%G!<%?$NJ,3d$r7h$a$k%Q%i%a!<%?$G$"$k(1) G [Ns<!8 5% "%i%$%s%a%s%H! "(2) %V%m%C%/%5%$%:Hf! "(3) %V%m%C%/%5%$%:! "(4) J,3d<! 8 5! "$r=g$K7hD j7F$$$/! #J,3d<!8 5$N7hD j$G$O! "J,3d<!8 5$N8uJd$r@8@ .$7! "3F8u JdKBP7$F@-G=8 +@Q$r9T$&$3HG! ":GE,$JJ,3d<!8 5$r7hD j9k$H$$$&% "%W%m!< %ArH$k! #

5i$K! "K\O@J8$GDs0F9k%G!<%?J,3d7hD j<jK !$r! "JBNs2=%3%s%Q%$%i **TINPAR** KBP7FE,MQ7I >2A$r9T$C$?7k2L$r< ($9! #

0J2<! "Bh2 >OGOGX7JH7$FK\8 &5 f$G<h$j0 7$&%G!<%?J,3d7hD jLdB jKD$$$F

=RY?>e$G!"4XO"8&5f$H$=$NLdBjE@$r;XE&$9$k!#Bh3>O$G$ODs0F$9$k%G!<%?J,3d7hDj<jK!$N>\:YKD$$$F=RYk!#5i$K!"Bh4>O$G$O!"Ds0F$9$k%G!<%?J,3d7hDj$N3F%9%F%C%WN&$A!"J,3d<!857hDj$N%9%F%C%W$GMxMQ$9$k@-G=8+@QJ}K!$N>\:Y$r=R$Y$?8e!"Bh5>O$GI>2A$r<($7!"Bh6>OG^Ha$k!#

2 %G!<%?J,3d7hDj$NJ}?K

2.1 JBNs2=%3%s%Q%$%i$H$N4X78

%a%C%;!<%88r497?JBNs7W;;5!$rMQ$$$F%G!<%?JBNs@-$r;}$D%W%m%0%i%`$r<B9T$9kK$O!"G[Ns%G!<%?$r%W%m%;%C%5$,8GM-$K;}$D%a%b%j$KJ,;6$7$FG[CV7J1lPJiJ$$!#%G!<%?J,3d7hDjHO!"3NG[Ns%G!<%?$NG[CV$r7hDj9k3HG"$k!#

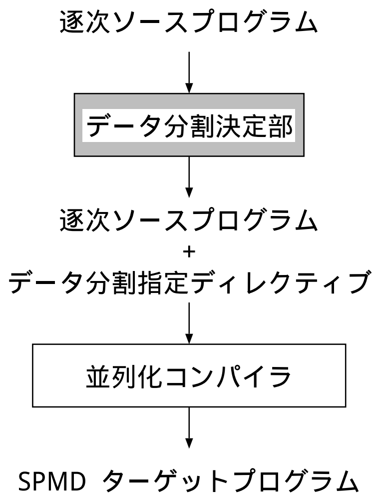
K\8&5f$,BP>]$H9kNO!"%a%C%;!<%88r497?$NJBNs7W;;5!8~$1$NJBNs2=%3%s%Q%$%iG"$j!"Owner Computes Rule KhCFJBNs2=$r9T$&3H$rA0Ds$H9k!#Owner Computes Rule HO!"BeF~J8$N1&JU$K8=lk1i;;$O!":8JU$N%G!<%?$r=jM-$9$k%W%m%;%C%5$,<B9T9k$H$$$&5,B'G"$k!#Nc$($PHPF (High Performance Fortran)[2] %3%s%Q%$%i$H8FPlkb$NB?$/O3$N<o$NJBNs2=%3%s%Q%$%i$G$"$k!#2f!9$,8=:_3+H/Cf$NTINPAR $bF1MM$K!"Owner Computes Rule $K=>$C$FJBNs2=$r9T$$!"%a%C%;!<%88r497?JBNs7W;;5!8~1NJBNs2=%3!<%I$r=PNO$9$k!#

=>Mhhj3N<o$NJBNs2=%3%s%Q%$%i$NB?$/$O!"%f!<%6$KhC$F%G!<%?J,3d$,7hDj5l!"%G%#%l%/%F%#%VH7$F%3%s%Q%$%i$K;X<($5lk3H$rA0Ds$H7F$$$?!#3lKBP7!"K\8&5f$NL\I8$G$"$k<+F0%G!<%?J,3d7hDjIt$O!"%f!<%6$Khk3liN;XDj$rBe9T$9$k!#$9Jo$A!"C`<!%=!<%9%W%m%0%i%`$rF~NOH7!"$=$lir2r@O7F5aa?%G!<%?J,3d%G%#%l%/%F%#%V$rC`<!%=!<%9%W%m%0%i%`$KIU2C$7!"$=li$rJBNs2=%3%s%Q%$%iKEO9!#JBNs2=%3%s%Q%$%i$OM?$($il?%G%#%l%/%F%#%VrbHK!"JBNs2=5l$?SPMD %W%m%0%i%`$r=PNO9k(?^2.1)!#

2.2 07$&%G!<%?J,3d

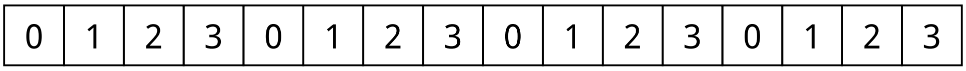
0l35$K!V%G!<%?J,3d!W$H8@CF$b!"$5$^$6$^$JJ,3dJ}K!$,9M$(il$k!#

Bh2.1@a$G=R$Y$?$h$&$JJBNs2=%3%s%Q%$%i$N$[$HsI$O!"2J3X5;=Q7W;;$Nh&$K!"%k!<%WJ8$Khk5,B'E*$J%G!<%?%"%/%;%9%Q%?!<%s$r;}$A!"=hM}FbMF$KB?$/$NJBNs@-$r;}$DbN$rBP>]$H$7!"$=li$r9bB.$K<B9T9k$?$a$K!"?^2.2$Nh&$J%V%m%C%/J,3d!"%5%$%/%j%C%/J,3d!"%V%m%C%/%5%$%/%j%C%/J,3d$H$$$C$?5,

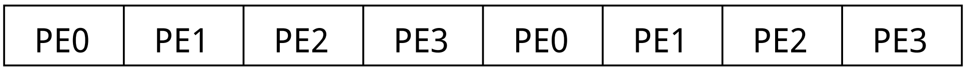


?^2.1: JBNs2=%3%s%Q%$%i$H$N4X78

(a)ブロック分割



(b)サイクリック分割



(c)ブロックサイズ2のブロックサイクリック分割

?^2.2: 5,B’E*$J%G!<%?J,3d

B’E*$JG[Ns%G!<%?J,3dJ}K!$rE,MQ$7!"JBNs@-$rCj=P9k!#$^$?!"5UK3li$N JBNs2=%3%s%Q%$%i$O!"$3li$N%G!<%?J,3d$KBP7FN_%3!<%I@8@.2DG=Jb$N $,$[HsIG$"$k!#3&$$$C$?M}M3$+$i!"B?$/$N%G!<%?J,3d7hDj<jK!$K4X9k 8&5fGO!"3liN5,B’E*$J%G!<%?J,3d$NCf$+$i3FG[Ns$N%G!<%?J,3d$r5aak!# K\8&5f$bF1MM$NM}M3Kh$j!">e5-$N5,B’E*$J%G!<%?J,3d$N$_$r9M$($k3HK9 $k!#JBNs2=%3%s%Q%$%iTINPAR$b!"$3$&$$$C$?5,B’E*$J%G!<%?J,3d$r%5%]!<%H 7F$$$k!#

2.3 @EE*%G!<%?J,3d7hDjLdBj

5F!"3liN5,B'E*$J%G!<%?J,3d$NCf$+$i%G!<%?J,3d$r5a$a$k:]$KLdBjHJkN$O!"%W%m%0%i%`$r2?i+$NC10L—Nc$($P!"%k!<%W$d!"%5%V%k!<%A%sJI—$GJ,$1$?3FItJ,(0J2<%U%'!<%:$H8FV3HK9k)KhCF:GE,HJ$k%G!<%?J,3d$,0[Jk(3l$r !VG!<%?J,3d$N6%9g!W$H8F$V)3H$,$"kH$$$&3HG"$k!#$3Nh$&$J!"%U%'!<%:4VGN%G!<%?J,3d$N6%9g$O!"%G!<%?J,3d7hDj$r:$FqK9$k860x$NRHDG$"$k!#3Nh&$J!"%U%'!<%:4V$N%G!<%?J,3d$N6%9g$KBP7F!"@EE*%G!<%?J,3d!"F0E*%G!<%?J,3d$N2DL$j$NBP=hK!$,9M$($ilk!#3liN$&$A!"K\O@J8GO!"@EE*%G!<%?J,3d$N7hDj<jK!$r<h$j07$&!#

@EE*%G!<%?J,3d$O!"%W%m%0%i%`$rDL7F3FG[NsJQ?tKBP9$k%G!<%?J,3d$rJQ99$;$:$K<B9T$r9k3H$rA0Ds$H7F5aailk%G!<%?J,3dG"$k!#$3$N>l9g!"%U%'!<%:4V$G6%9g9k%G!<%?J,3dN&$A1$D$rA*$S!"$=$NJ,3dG9YF$N%U%'!<%:$r<B9T9k!#3N$?$a!"7hDj5l$?J,3d$,:GE,HJiJ$$%U%'!<%:$,B8:_$7!"3liN%U%'!<%:$N<B9T%3%9%H$OA}2C9k!#=>CF!"%W%m%0%i%`A4BNH7F3li$N%U%'!<%:$N<B9T%3%9%H$NA}2C$,:G>.HJkh$&$J%G!<%?J,3d$r7hDj$9kI,MW,$"$k!#

3lKBP7!"F0E*%G!<%?J,3d$r9T$&>l9g$O!"%U%'!<%:4V$G%G!<%?$N:FJ,3d(redistribution)$r5v$9$3HrA0DsH7$F5a$ail$k%G!<%?J,3d$G$"$k!#3N>l9g!":FJ,3d$,5v$5lkJ,!"@EE*%G!<%?J,3dhj$b2DG=$J%G!<%?J,3d$N<dN`$,A}$($k$?$a!"$h$j:GE,$J%G!<%?J,3d$,5a$a$i$l$k2DG=@-$,$"$k!#$7$+$7!"%W%m%0%i%`A4BN$H7F:GE,$J%G!<%?J,3d$r5aak$?$aKO!"3F%U%'!<%:$G2DG=$J%G!<%?J,3d9YFNAH$_9g$o$;$KBP7F!"3F%U%'!<%:GN<B9T;~4V$H!"3F%U%'!<%:4V$GI,MWHJ$k:FJ,;6$K$+$+$k;~4V$NOB$,:G>.$HJkh&JAH_9go;$r5a$akI,MW,$"$j!"@EE*%G!<%?J,3d$HHf3S$7$F!"C5:w6u4V$,GzH/E*$K3H$,$k!#

=>CF!":GE,$JF0E*%G!<%?J,3d$r5aak$?$aKO!"B?Bg$J;~4V$,I,MWHJ$j!"%3%s%Q%$%i$N0lIt$H7FAH$a9~$`3H$O:$FqG"$k!#KB:]!"D System GO!"3lir%G!<%?J,3d7hDj;Y1g%D!<%kH7$F<B8=$;6k$rF@$J$$>uBVG"$k[3, 4]!#

K\8&5fGO!"8=<BE*$J;~4VFb$K<+F0E*$K%G!<%?J,3d$r5aak3H$rL\E*$H9k$?$a!"@EE*%G!<%?J,3d$r:NMQ$7$?!#$5iKIU$12C$(kH!"F0E*%G!<%?J,3d$N3F%U%'!<%:$G$N%G!<%?J,3d$N8uJd$r9J$kNK!"@EE*%G!<%?J,3d7hDj$N5;=Q$,I,MWG"k?$a!"@EE*%G!<%?J,3d7hDj$O!"F0E*%G!<%?J,3d7hDjLdBj$N4pAC$HJk=EMW$J8&5f$G$"$k$H8@$($k!#

2.4 %G!<%?J,3d7hDjK*1k@-G=8+@Q$NLr3d

K\<jK!GO!"%G!<%?J,3d$N7hDj$N4 DN%9%F%C%WN&$A!":G=*%9%F%C%W$G$"$kJ,3d<!85$N7hDj$K$*$$$F!"@-G=8+@QjrMxMQ9k!#

JBNs2=%3%s%Q%$%i$K$*$1$k@-G=8+@Q$j$O!"0lHIE*$K:GE,2=$N8z2L$rDjNIE*$KH=CG$9$k<jCJ$H7FMQ$$$i1k3H$,B?$$!#Nc$($P!"%k!<%W%$%s%?!<%A%'%s%8$rE,MQ9Y$-$+I&$+$NH=CG!"%a%C%;!<%8%Y%/%H%k2=$N%Y%/%H%k2=D9$N7hDjJI$KMQ$$$i1k!#$^$?!"%5%V%k!<%A%s$d!"%k!<%W$JIrC10LH9$k%U%'!<%:4V$G$N%G!<%?J,3d$r7hDj9k:]Kb!"@-G=8+@QjKhk<B9T;~4V$NM=B,$,H=CG4p='H7$FMxMQ$5$1$k!#

@EE*%G!<%?J,3d7hDjK*$$$F$b!"@-G=8+@Q$jKh$kH=CG$,I,MWHJ$k!#$J$<$JiP!":GE,$J%G!<%?J,3d$r5aak$?$aKO!"3F%G!<%?J,3dKhCFCj=P$5$1$kJBNs@-$d!"1i;;$HDL?.$N%P%i%s%9JI$r9MN8$9kI,MW,$"$j!"$3$1i,:GE,2=$5$1$F$$$k$+$I$&$+$NH=CG$,I,MWKJk?aG$"$k!#

FC$K!"J,3d<!85$N7hDj$O!"JBNs2=%3%s%Q%$%iKhCFCj=P$5$1$kJBNs@-$KBg$-$J1F6A$rM?$(k?$a@-G=8+@Q$,I,MWG"$k!#

2.5 @EE*%G!<%?J,3d$K4X$9$k4XO"8&5f

@EE*%G!<%?J,3d$r5a$a$k4XO"8&5f$H7F!"JBNs2=%3%s%Q%$%i**PARADIGM**[7] $N%G!<%?J,3d7hDj%U%'!<%:$,$"$k!#$^$?!"@EE*%G!<%?J,3d$O!"FOE*%G!<%?J,3d$r7hDj9kNKbI,MWJ5;=QG"$j!"**D System** GO3F%U%'!<%:4H$K@EE*%G!<%?J,3d$N8uJd$r5a$a$k!#$=3G!"$3$1$i2 DN8&5f$HK\8&5fHrHf3S$7!"2f!9$N<jK!$NFCD'$r=RYk!#

PARADIGM HNHf3S Illinois Bg3X$G3+H/$5$1$F$$$kJBNs2=%3%s%Q%$%i**PARADIGM** $O!"K\8&5f$H$[$\F1MM$NJ}K!$KhC$F@EE*%G!<%?J,3d$r7hDj9k!#H'Ey$N<jK!$G$O!"%W%m%0%i%'$N%U%'!<%:4V$G6%9g$9$kJ,3d$NCf$+$i1 DNJ,3d$r5a$ak?aKMxMQ$5$1$k%0%i%U$N%(%C%8$N=E$_H7$F!"@-G=8+@Q$N7k2L$rMxMQ$9$k!#%G!<%?J,3d$N3F%Q%i%a!<%?$N7hDj$N2aDx$G@-G=8+@Q$r9T$&$N$G!"B>$NJ,3d$N%Q%i%a!<%?$,LDjN$^$^G"$k!#$7$+$7!"@-G=8+@Q$O%G!<%?J,3d$,9Y$FM?$(i1$J$$$HG-$J$$$N$G!"H'Ey$N<jK!GO!"LDjN%Q%i%a!<%?O9YF$"$i$+$8aDjai1$?CM$G$"$kH7$F8+@Q$i1k!#3N$?$a!"J#;($J=hM}$rI,MWH9$k@-G=8+@Q$jr9kKb+$+$oi:!"$=$1$[$I@53N$J>pJs$OF@i1$J$$!#

0lJ}!"K\0@J8$GDs0F$9$k<jK!$O3N%0%i%U$N%(%C%8$N=E$_$H7FBeF~J8$N<B

9T2s?t$r;H$&$?$a!"C1=c$JJ}K!$KhC$F5a$ak3H,G-$k!#

$^$?!"J,3d<!85$r5a$a$kJ}K!$O2f!9$N<jK!$HF1MM$K!"J,3d$N8uJd$r@8@.$7!"3F8uJdKBP7$F@-G=8+@Q$j$r9T$&$H$$$&J}K!rH$k!#2f!9$N<jK!Gb9G$K=R$Y$?DL$j!"@-G=8+@QjrMxMQ7FJ,3d<!85$r5a$a$k!#H`Ey$N%"%W%m!<%AHN0c$$$O!"@-G=8+@Q$,2DG=$JHO0O$r9J$k3HKh$j!"8+@Q$jrh$j@53N$K7h$&$H9kE@G"$k!#$h$j@53N$K8+@Qk3HKhj!"hj:GE,$J%G!<%?J,3d$r5aak3H$,2DG=$KJk3H$,4|BT$G$-$k!#$^$?!"8+@Q$,IT2DG=$J>l9gKBP7FbG[Ns<!85J,3dE,@-EYKh$k<jK!$KhC$FJ,3d<!85$r5aak3H$,2DG=$G$"$k!#
D System$H$NHf3S Rice Bg3X$N**D System**$G$O!"F0E*%G!<%?:FG[CV$r9T$&$Y$-$+$r9MN8$KF~lF%G!<%?J,3d$r7hDj$9$k!#$3$N<jK!$G$O!"$^$:%W%m%0%i%`$r%k!<%W$rC10L$H7F%U%'!<%:$KJ,$1!"$=$N%U%'!<%:4H$K@EE*%G!<%?J,3d$N8uJd$r$$$/D+5aak!#3F%U%'!<%:$N@Z$lL\$G%G!<%?$N:FG[CV$r9k>l9gH7$J$$>l9g$N<B9T%3%9%H$r@-G=8+@QjKhC$F5a$a!"$=$liNAH$_9g$o$;$NCf$+$i:GE,$J%G!<%?J,3d$r5aak!#**D System**$,3F%U%'!<%:$KBP9k@EE*%G!<%?J,3d$O!"K\<jK!$HF1MM$K%0%i%U$r:n@.7F5aak!#%(%C%8$N=E$_E1$O!"**PARADIGM**$HF1MM!"@-G=8+@QjKhC$F5a$a$^$kDL?.%3%9%HrD1k!#3lb5$-$[$I;XE&$7$?LdBj$r4^sG$$$k!#

$^$?!"H`Ey$N<jK!$O!"9-$$C5:w6u4VKBP7$FC5:w$r9T$&$N$G!"%G!<%?J,3d7hDj$K;~4V$,$+$+$k!#3N$?$a!"%3%s%Q%$%i$N0lItH7$FAH9~$^$:!"%G!<%?J,3d7hDj$N$?$a$N;Y1g%D!<%k$H7FDs6!5l$F$$$k$?$a!"<+F0JBNs2=$OC#@.$G$-$F$$$J$$$H8@$($k!#

3 %G!<%?J,3d7hDj<jK!

3.1 %G!<%?J,3d7hDjN?aNI>2A9`L\

JBNs<B9T;~$K@-G=$KBg$-$J1F6A$rM?$($k9`L\$H7F0J2<$N3$D$,9M$(il$k!#

1. %W%m%;%C%54V$NDL?.NL
2. Cj=P5l$kJBNs@-
3. %W%m%;%C%54V$NIi2Y%P%i%s%9

3liN3F9`L\$KBP1~$9$kNL$O!"%G!<%?J,3d$K$h$C$FBg$-$/JQ2=$9$k!#$3$N$?$a!"Ds0F$9$k<jK!$G$O!"$3$l$i$N3F9`L\$r:GE,2=9k3H$rL\E*$H9k!#0J2<$K!"%G!<%?J,3d$H3F9`L\$H$N4X78$K$D$$$F=RYk!#

%W%m%;%C%54V$NDL?.NL %a%C%;!<%88r497?JBNs7W;;5!GO!"$"$k%W%m%;%C%5$,1i;;$KI,MW$J%G!<%?$r=jM-7F$$$J$+$C$?>19g!"DL?.$KhC$F%G!<%?$r;2>H7J1lPJiJ$$!#7?$,$C$F!"DL?.NL$O%G!<%?J,3dKhCF0[JCF/$k!#DL?.$Khk%G!<%?$N;2>H$O!"%m!<%+%k$K;}$D%G!<%?$N;2>H$KHfYF%3%9%H$,Bg$-$$$?$a!"9bB.2=$N$?$aKODL?.NL$r2DG=$J8B$j8:$i$9I,MW$,$"$k!#

Cj=P5l$kJBNs@- Owner Computes Rule $K=>$C$?JBNs2=$r9T$&>19g!"%G!<%?J,3d$r7hDj7?CJ3,$G!"$I$N=hM}$,IN%W%m%;%C%5$G<B9T$5lk$+$,7h$^$CF7$^$&!#7+7J$,$i!"=hM}4V$K0MB84X78$,$"$k>19gKO!"$=$liN4X78$rJ]B8$7$?<B9T$,I,MWG"$j!"=hM}$N3djEvFJ}$,0-$$$HJBNs@-$,Dc2<$7F7$^$&!#=>CF!"=hM}$,$INh$&$KJ,3d5l!"$=$liN=hM}4VKINh$&$J0MB84X78$,$"kN$+$r9MN8$7$F%G!<%?J,3d$r5a$akI,MW,$"$k!#

%W%m%;%C%54V$NIi2Y%P%i%s%9 Ii2Y$,6Q9U7F$$$J$$$H!"3F%W%m%;%C%5$,7W;;$r9T$&;~4V$N!"A4BN$N<B9T;~4V$KBP9k3d9g$,Dc2<$9k?$a!"Ii2Y$N6Q9U2=$O@-G=8~>e$N$?$a$N=EMW$JMW0xG"$k!#0lJ}!"Owner Computes Rule $K=>$C$?JBNs2=$G$O!"%G!<%?J,3d$KhC$F%W%m%;%C%5$X$N=hM}$N3djEvF$,7h$^k?$a!"3d$jEvFilk=hM}NL$b%G!<%?J,3d$KhC$F7h$^$k!#=>$C$F!"3F%W%m%;%C%5$K3djEvFil$k=hM}NL$r9MN8$7$F%G!<%?J,3d$r5a$akI,MW,$"$k!#

3.2 %G!<%?J,3d7hDj$N35MW

3.2.1 %G!<%?J,3d7hDj$N<j=g

%G!<%?J,3d7hDj$O!"0J2<$N4DN%9%F%C%WKhCF9=@.5l$k(?^3.1)!#

$^$?!"3F%9%F%C%WK*$$$F$O!"Bh3.1@a$NJ}?KG"2?I>2A9`L\$KN10U7F7hDj$r9T$&!#

1. G[Ns<!85%"%i%$%s%a%s%H$N7hDj

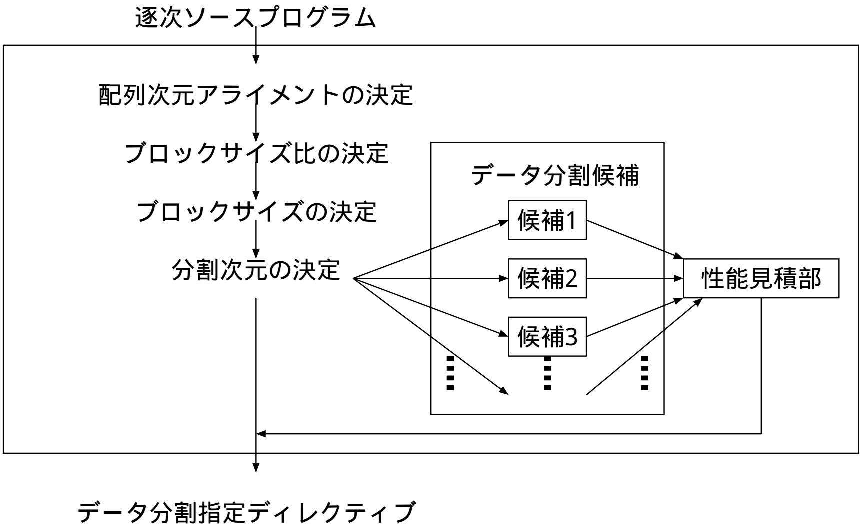
   9YFNG[Ns<!85NAHKBP7F!"$=$liN<!85$NJ,3d$NM-L5$rF1$8K9Y-$+$I$&$+$H$$$&4X78$r2r@O$7!"$=$liN4X78$+$i!"G[Ns<!85$r%0%k!<%WJ,$1$9$k!#3N:]!"%W%m%;%C%54V$NDL?.NL$r9MN8$9$k!#

2. %V%m%C%/%5%$%:Hf$N7hDj

   G[NsJQ?t$N3F<!854V$N%V%m%C%/%5%$%:Hf$N@)Ls$r5a$a$k!#$3$N:]!"%W%m%;%C%54V$NDL?.NL$r9MN8$9$k!#

3. %V%m%C%/%5%$%:$N7hDj

   >e5-$N3F%V%m%C%/%5%$%:Hf$+$i!"<B:]$K%V%m%C%/%5%$%:$r5a$a$k!#$3$N:]!"



?^3.1: %G!<%?J,3d7hDjIt$NFbIt9=@.

```
DO j = 0 , n - 1
  DO i = 0 ,n - 1
    A(j,2*i) = 𝓕(B(3*i,j))
  ENDDO
ENDDO
```

?^3.2: G[NsJQ?t4V$NBeF~J8

%W%m%;%C%54V$NDL?.NL$HIi2Y%P%i%s%9$r9MN8$9$k!#

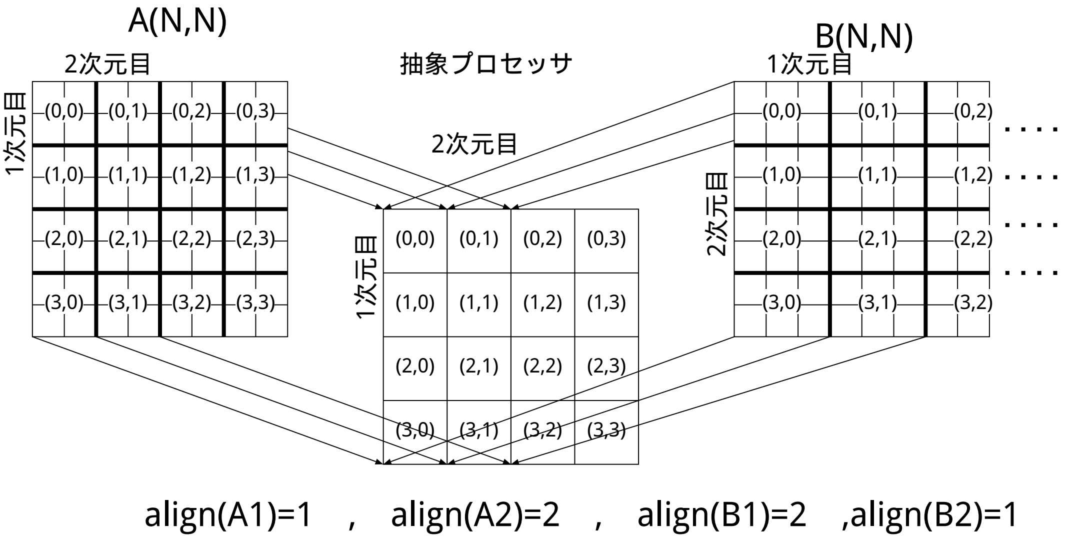
4. J,3d<!85$N7hDj

C1>]%W%m%;%C%5NI$N<!85$rJ,3d9k$+$r7hDj9k!#%W%m%;%C%54V$NDL?.NL!"Cj=P$5lkJBNs@-$r9MN8$9k?aK!"J,3d<!85$N8uJd$r@8@.$7!"3F8uJd$KBP$7$9$k@-G=$r8+@Qk3HKhj!"J,3d$NM%Nt$rH=CG9k(?^3.1)!#

3.2.2 J,3d7hDj$N3F%9%F%C%W$N=g=x

%G!<%?J,3d7hDj$rA0@a$G=RY?4 DN%9%F%C%WKhCF9=@.9kM}M3$r=R$Y$k!#

$^$:!"NcH7$F!"?^3.2 Nh$&$J%W%m%0%i%`$NCGJR$K$D$$$F9M$($k!#1&JU$N $\mathcal{F}$(B(3*i,j)) $O!"G[NsMWAGB(3*i,j) $r4^$`<0G"k3HrI=$9!#3N%W%m%0%i%`$KBP7F$O!"$Hj(k9YF$Ni,j NCMKBP7F!"2 DNG[NsJQ?tMWAG A(j,2*i)!"B(3*i,j) $,$9YFF1$8%W%m%;%C%5$KJ,3d5l$F$$$l$P!"DL?.NL$O:GE,2=5l$k!#$b$7!"A $N1<!85L\$,J,3d5l$k>l9g$O!"B $O2<!85L\$rJ,3d$7!"$=l>lNMWAG$r%W%m%;%C%5$KBP7FF1$8NL$:$D3d$jEvFl$P!"DL?.$,H/@8$7



?^3.3: ?^3.2 $N%W%m%0%i%`Cf$NG[NsJQ?t$NCj>]%W%m%;%C%5$X$N3d$jEv$F

$J$$J,3dHJ$k!#F1MM$K!"A $N2 <!85L\$,J,3d$5$1$k>19g$O!"B $O1 <!85L\$rJ,3d$7!"$=1>1NMWAG$N%W%m%;%C%5$X$N3d$jEvFHfr2 : 3 H9$1$P!"DL?.$,H/@8$7$J$$J,3dHJ$k!#

3Nh&$K!"G[NsJQ?t$I$&$7$NBeF~J8$r2r@O9k3HKhCF!"G[NsJQ?t4V$NJ,3d$K4X9kAjBPE*$J4X78$,5aailk!#$=$3$G!"$3li$N4X78$rI=8=9k$?$a$K!"Cj>]%W%m%;%C%5$H$$$&35G0$rF3F~$9$k!#Cj>]%W%m%;%C%5$O!"%W%m%0%i%`Cf$K=P8=$9$kG[NsJQ?t$N:GBg<!85?t$HF1$8<!85?t$G9=@.$5$l!"2r@OBP>]$H$9$k%W%m%0%i%`KD$-1 $DMQ0U$9$k!#$3lKBP$7!"J*M}E*$KB8:_9k%W%m%;%C%5$r<B%W%m%;%C%5$H8F$V!#

Cj>]%W%m%;%C%5$r;H$&$H!"G[NsJQ?t4V$NJ,3d$K4X$9$kAjBPE*$J4X78$O!"3FG[NsJQ?t$r!"Cj>]%W%m%;%C%5$N$$$:l+$NMWAG$X$N3d$jEvFH7FI=8=G-$k!#?^3.2 NNcG$O!"A $N1 <!85L\$HB $N2 <!85L\$r@0Ns$7!"3FG[Ns$N1 MWAG$rCj>]%W%m%;%C%5$N1 MWAG$K3d$jEv$F!"$5iK!"A $N2 <!85L\$HB $N1 <!85L\$r@0Ns$7!"A $O2 MWAG$:$D!"B $O3 MWAG$:DrCj>]%W%m%;%C%5$N1 MWAG$K3djEvF$k(?^3.3)!#$3$N8e!"Cj>]%W%m%;%C%5$r<B%W%m%;%C%5$KJ,3d$9k3HKhj!"G[NsJQ?t4V$NJ,3d$K4X9kAjBPE*$J4X78$OJ]$?$l!":GE,$J%G!<%?J,3d$H9k3H$,$G$-$k!#

A0@a$G=R$Y$?4 DN%9%F%C%WN&$A!"G[Ns<!85%"%i%$%a%s%H$N7hDj$O!"G[NsJQ?t$NCj>]%W%m%;%C%5XN3d$jEvFN$&$A!"G[Ns<!85$HCj>]%W%m%;%C%5$N<!85$NBP1~IU$1r9k3HKAjEv9k!#$^$?!"%V%m%C%/%5%$%:Hf$N7hDj$O!"3FCj>]

%W%m%;%C%5$N<!85$4HK!"$=$3$K3d$jEvFil?G[Ns<!85$NMWAG$r2?8D3djEvFkN$+$r5aak3H$KAjEv$9$k!#%V%m%C%/%5%$%:Hf$N7hDj$O!"3FG[Ns<!85$,Cj>]%W%m%;%C%5$N<!85$KBP1~IU$1ilF+iG$J$$$H5a$^$i$J$$!#=>CF!":G=i$KG[Ns<!85%"%i%$%a%s%H$r5a$a!"<!$K%V%m%C%/%5%$%:Hf$r5a$a$k!#

$^$?!"Cj>]%W%m%;%C%5$N<B%W%m%;%C%5$X$NJ,3d$O!"G[NsJQ?t$N<B%W%m%;%C%5$X$NJ,3d$HF1MM$NJ}K!$G9T$&!#$9Jo$A!"Cj>]%W%m%;%C%5$N3F<!85$rJ,3d$9k+$r7hDj$7!"J,3d9k<!85KBP7FOJ,3d$N%V%m%C%/%5%$%:$r7hDj$9$k!#K\<jK!$G$O!"J,3d<!85$r7hDj9k:]!"J,3d<!85$4$HK=li$NM%Nt$r@-G=8+@QKhCFHf3S9k!#@-G=8+@QjO!"J,3d$,3NDj$5lF$$$J1lPG$-$J$$$?$a!"J,3d<!85$4HN@-G=$NHf3S$r9T$&:]$K%V%m%C%/%5%$%:$,7h$^$C$F$$$kI,MW$,$"$k!#=>$C$F!"Cj>]%W%m%;%C%5$N9YFN<!85$N%V%m%C%/%5%$%:$N7hDj$r9TC?8e!"J,3d<!85$r5a$a$k!#K\<jK!$Khk%V%m%C%/%5%$%:$N7hDj$O!"8e$G>\$7$/=R$Ykh$&$K!"G[NsMWAG?t$HA0%9%F%C%W$G5a$^$C$?%V%m%C%/%5%$%:Hf$+$i5aak$?$a!"Cj>]%W%m%;%C%5N9YF$N<!85$KBP7F%V%m%C%/%5%$%:$r5aakKOJ#;($J=hM}$OI,MW$J$$!#

3.2.3 %G!<%?J,3d$N%Q%i%a!<%?

K\@aGO!"3F%9%F%C%W$G7hDj$9Y-3H$r%Q%i%a!<%?2=$7!"L@3N2=9k!#

G[NsMWAG$NCj>]%W%m%;%C%5$X$N3d$jEv$F　$3$3GN$ <!85G[NsJQ?t(A $H8F$V) $N3FG[NsMWAG$NCj>]%W%m%;%C%5XN3djEvFJ}$D$$$F=RYk!#%W%m%0%i%`Cf$K8=$l$kG[NsJQ?t$N:GBg<!85?trD$ H9kH!"Cj>]%W%m%;%C%5OD$ <!859=@.rHk3HKJk!#$^$?!"G[NsJQ?tA NBhk$ <!85L\$r$\mathtt{A}_k$$H=q$/$3HK9k!#

$3$3$G!"G[NsJQ?t$N<!85$+$iCj>]%W%m%;%C%5$N<!85$X$N4X?t`align` $rDj5A$9$k!#4X?t`align` KhCF!"G[Ns<!85$,Cj>]%W%m%;%C%5$N<!85N1$ DKBP1~E1il$k!#$3$NBP1~$E1,!"G[Ns<!85%"%i%$%s%a%s%H$N7hDj$KAjEv$9$k!#`align` 4X?t$O!"0J2<$N@-<A$KhC$FFCD'$E$1$5lk!#

- `align`($\mathtt{A}_k$) $NCM0h$O!"=89g$\{1, 2, \cdots, D\}$ G"$k!#
- F1$8G[NsJQ?t$KB0$9$k0[JkG[Ns<!85$O!"%W%m%;%C%59=@.$N0[Jk<!85$KBP1~$E1ilk!#9JoA!"$1 \le k \le N, 1 \le j \le N$ JkG$0U$N@0?tk, j KBP7$F!"$k \neq j$ Ji$P!"`align`($\mathtt{A}_k$) $\neq$ `align`($\mathtt{A}_j$) G"$k!#

G[Ns<!85%"%i%$%s%a%s%H$O!"Cj>]%W%m%;%C%5$N3F<!85$KBP7FG[Ns<!85$,%0%k!<%WJ,$1$5$l$?$H8+k3HbG-kN$G!"K\O@J8$G$OG[Ns<!85%"%i%$%s%a%s%H$HG[Ns<!85$N%0%k!<%WJ,1rF1$80UL#$GMQ$$$k3HK9$k!#

<!$K!"Cj>]%W%m%;%C%5$N3F<!85J}8~KD$$$F!"Cj>]%W%m%;%C%5$N1MWAG$KBP7FG[NsJQ?t$N2?MWAG$r3djEvFk+$r7h$a$k!#$9Jo$A!"Cj>]%W%m%;%C%5$N align(A_k) <!85L\J}8~$N1MWAG$KBP9k!"G[Ns<!85A_kJ}8~$NMWAG$N3djEvF8D?t$\alpha_{\mathtt{A}_k}$$r5aak!#3l$O%V%m%C%/%5%$%:Hf$N7hDj$KBP1~9k!#

Cj>]%W%m%;%C%5$N<B%W%m%;%C%5$X$NJ,3d Cj>]%W%m%;%C%5$r<B%W%m%;%C%5$XJ,3d$9kK$"$?$j!"Cj>]%W%m%;%C%5$NIN<!85$rJ,3d$9kN$+$r5aak!#3l$,J,3d<!85$N7hDj$N%9%F%C%W$KBP1~9k!#

5i$K!"Cj>]%W%m%;%C%5$NBhk<!85L\$(k = 0, 1, \cdots, D)$$,J,3d5l$k>l9g!"$=$N<!85J}8~$NBhiHVL\$NMWAG$,3d$jEvFilk<B%W%m%;%C%5HV9f$r5a$akI,MW,$"$j!"3l$O<!$NJ,3d4X?t$f_k$$KhCFDjail$k!#

$$f_k(i) = \left\lfloor \frac{i}{b_k} \right\rfloor [\bmod\ P_k] \tag{3.1}$$

$3$3$G!"$b_k$$O!"Cj>]%W%m%;%C%5NBhk$<!85L\$NJ,3d$N%V%m%C%/%5%$%:$rI=$7!"P_k $OCj>]%W%m%;%C%5$NBhk<!85L\$K3djEvFil$k<B%W%m%;%C%5?t$G$"$k!#3NJ,3d4X?tN&$A!"$b_k$$N7hDj$,%V%m%C%/%5%$%:$N7hDj$KBP1~9k!#

$[\bmod\ P_k]$ $O!">JN,2DG=$G$"$j!"3l$,>JN,$5l?>l9g$O%V%m%C%/J,3d!"$b_k$$,1 $N>l9g$O%5%$%/%j%C%/J,3d!"$=$l0J30$N>l9g$,%V%m%C%/%5%$%/%j%C%/J,3d$KAjEv$9$k!#$?$@$7$3$liOCj>]%W%m%;%C%5KBP9$k<B%W%m%;%C%5$X$NJ,3d$G$"$j!"<B:]$NG[Ns<!85$A_k$$KBP7F$O!"1 Cj>]%W%m%;%C%5$K$\alpha_{\mathtt{A}_k}$8D$NMWAG$,3djEvFil$F$$$kNG!"1 DN<B%W%m%;%C%5Nk$ <!85L\$K$O!"$b_k \times \alpha_{\mathtt{A}_k}$8D$NG[NsA $NMWAG$,3djEvFil$k!#

3.3 G[Ns<!85%"%i%$%s%a%s%H$N7hDj

G[Ns<!85%"%i%$%s%a%s%H$N7hDj$O!"align 4X?t$,F1$8CM$rHkh&$JG[Ns<!85$N=89g$r!"$=NCM4HK5aak3H$KAjEv$9$k!#$3li$N7hDj$K$"$?$j!"%G!<%?J,3d7hDj$NJ}?K$G=R$Y$?%a%C%;!<%8DL?.NL$r:GE,K9k3Hr9MN8$9$k!#

K\%9%F%C%WGO$^$:!"C`<!%=!<%9%W%m%0%i%`$K8=$olkG[NsJQ?tI&7NBeF~J8$r2r@O$9$k!#<!$K!"3N2r@O>pJs$+$i!"**CAG**(Component Affinity Graph)[6] $H8F$Plk%0%i%U$r:n@.$7!"$=$l$rMQ$$$FG[Ns<!85%"%i%$%s%a%s%H$r7hDj9k!#0J2<!" CAG $,2?$rI=8=7?bNG"k+!"%a%C%;!<%8DL?.$r:GE,$K9kG[Ns<!85%"%i%$%s%a%s%H$,CAG KhCF$$$+$K5a$ailk+KD$$$F=RYk!#

3.3.1 G[Ns<!85%"%i%$%s%a%s%H$H**CAG**

G[NsJQ?tI&7NBeF~J8$N2r@O$K$*$$$F!"G[NsJQ?t$NE:;z<04V$N%"%U%#%s4X78[6] $H$$$&bN$KCmL\$9$k!#K\O@J8$G$O!"%"%U%#%s4X?t!"%"%U%#%s4X78$H$$$&MQ8l$r0J2<$N0UL#$GMQ$$$k3HK9$k!#

- %"%U%#%s4X?t— $f(i) = a \times i + b$ $N7A$G$"$io;$k4X?t$f$!#$?$@$7!"a, b $O@0?t$G$"$j!"$a \neq 0$!"i $O!"%k!<%WJQ?t$G$"$k!#
- %"%U%#%s4X78— JQ?t(>e$N4X?t$f$ GOi) $,F1$8%"%U%#%s4X?tI&7r!"8_$$$K%"%U%#%s4X78K"kH$$$&!#

$^$:BeF~J8$N:8JU$K8=ol$kG[NsJQ?t$H1&JU$K8=$olkG[NsJQ?tN=l>lN<!85$NE:;z<0$NAHKD$$$F!"%"%U%#%s4X78$NM-L5$rD4Yk!#

E:;z<0$N4V$K%"%U%#%s4X78$,$"C?>l9g!"<!Nh$&$JJ,3d$,$G$-$l$P!"$7J+C?>l9gKHfY$FDL?.NL$,8:$k!#$3lO!"Bh3.2.2@aG"2?NcNDLjG"$k!#

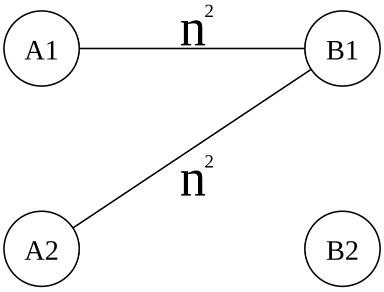
1. E:;z<0$,%"%U%#%s4X78$K$"$k<!85$O!"Cj>]%W%m%;%C%5$NF1$8<!85$K3djEvF$k!#
2. %V%m%C%/%5%$%:Hf$O!"E:;z<0$N%k!<%WJQ?t$N78?t($f(i) = a \times i + b$ KBP7FOa) NHfH9k!#(3N@-<A$rMxMQ$7$?J,3d$N7hDj$O!"%V%m%C%/%5%$%:Hf$N7hDj$N%9%F%C%W$G9T$&!#)

7+$7!"I,$:7b3Nh&$K:GE,$JG[Ns<!85%"%i%$%s%a%s%H$,5a$^$ko1GOJ/!"G[Ns<!85%"%i%$%s%a%s%H$K6%9g$,@8$8$k>l9g$b$"$k!#Nc$($P!"?^3.4$N%W%m%0%i%`$NNcGO!"B_1 OA_1$ HA_2$ $NN>J}$H%"%U%#%s4X78K"$k!#$3$N>l9g!"$3$NBeF~J8$GF1;~KI,MWH5l$kG[Ns$MWAG9YF$rCj>]%W%m%;%C%5$NF1$8MW$AG$K3d$jEvFkG[Ns<!85%"%i%$%s%a%s%H$OB8:_7J$$!#$^$?!"?^3.6GO!"1$DL\$N%k!<%W%\%G%#$NBeF~J8$H2$DL\$N%k!<%W%\%G%#$NBeF~J8$G$O!"$=l>lr6I=jE*$K8+$?>l9g$N:GE,$JG[Ns<!85%"%i%$%s%a%s%H$OO[Jk!#0J>e$N2$DNNcG$O!"G[Ns<!85%"%i%$%a%s%H$N2$D$N8uJd$,6%9g7F$$$k$,!"@EE*%G!<%?J,3d$r9k>l9g$O!":G=*E*$KIAi+$N8uJd$r:NMQ7J1lPJiJ$$!#

3Nh&$K!"G[Ns<!85%"%i%$%s%a%s%H$N8uJd$,6%9g7F$$$k>l9g!"%a%C;!<%8DL?.NL$r:GE,2=$9kH$$$&4QE@$+$i!"%W%m%0%i%`A4BNH7$F:GE,$JG[Ns<!85%"%i%$%s%a%s%H$r5aaJ1lPJiJ$$!#3N$?$a!"G[NsJQ?tI&7NBeF~J8$r2r@O$7$?>pJs$+$i**CAG**$r:n@.$7!"6%9g8uJd$+$i1$D8uJd$rA*Br$9$kLdBj$r%0%i%U$rMxMQ$7$F2r$/NG$"$k!#

```
DO j = 0 , n - 1
  DO i = 0 , n - 1 do
    A(i,i) = 𝓕(B(i,j))
  ENDDO
ENDDO
```

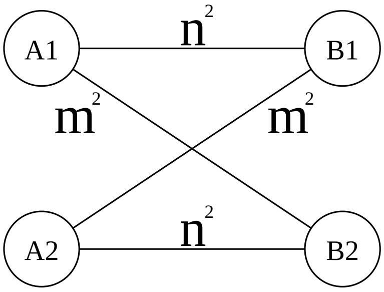
?^3.4: F1$8BeF~J8$G$N6%9g$NNc



?^3.5: ?^3.4 KBP9$kCAG

```
DO j = 0 , n - 1
  DO i = 0 , n - 1
    A(i,j) = 𝓕(B(i,j))
  ENDDO
ENDDO
       ⋮
DO j = 0 , m - 1
  DO i = 0 , m - 1
    A(i,j) = 𝓕(B(j,i))
  ENDDO
ENDDO
       ⋮
```

?^3.6: 0[JkBeF~J84VGN6%9g$NNc



?^3.7: ?^3.6 KBP9$kCAG

3.3.2 CAG $N:n@.J}K!

CAG $N%N!<%I$OG[NsJQ?t$N<!85$KBP1~9k!#%=!<%9%W%m%0%i%`$NBeF~J8$N2r@O$N7k2L!"0[$J$kG[NsJQ?t$N<!854V$K%"%U%#%s4X78$,$"$C$?>l9g!"$=$l$>$l$N<!85$KBP1~$9$k%N!<%I$N4V$K%(%C%8$,$D$/$i$l$k!#?^3.5!"?^3.7 $K$=$l$>$l?^3.4!"?^3.6 $N%W%m%0%i%`$NCAG $r<($9!#

%(%C%8$N=E$_ %k!<%W%\%G%#FbK"$kBeF~J8$O!"$=$N%k!<%W$,<B9T$5lk2s?tKhj=3+$iF@$ilkE:;z<0$N%"%U%#%s4X78$N;}$D=EMW@-$,0[Jk!#?^3.6 $N%W%m%0%i%`$G$O!"%k!<%W$N<B9T2s?t$rD j$a$kD j?tm!"n $NBg>.$KhC$F!"$=l>lN%\%G%#Fb$NBeF~J8$N<B9T2s?t$,0[$J$k!#$b$7!"m<n G"lP!":G=i$N%k!<%W%\%G%#Fb$NBeF~J8$N<B9T2s?t$NJ}$,B?$$!#3N>l9g!":G=i$NBeF~J8$N2r@O$+$iF@il$kG[Ns<!85%"%i%$%s%a%s%H$rM%@h$5$;$kY-G"$k!#$J$<$JiP!"<B9T2s?t$,B?$$$H$$$&3H$O!"$=$NBeF~J8$KhC$F7h$ail$k%"%/%;%9%Q%?!<%s

$K=>$C$?%G!<%?$N0\F0$,B?$$$H$$$&3HKJ$k!#$=N?$a!"$3$N2r@O$+$iF@$ilkG[Ns<!85%"%i%$%s%a%s%H0J30$N8uJd$,:NMQ$5l?>l9g!"%a%C%;!<%8DL?.NL$,B?$/Jk$+$iG"$k!#=>$C$F:GE,$J%"%i%$%s%a%s%H$r7hDj9k$?$aKO!"BeF~J8$N<B9T2s?t$r9MN8$KF~$lJ1lPJi$J$$!#

$=$3$G!"K\<jK!$G$O!":GE,$J%"%i%$%s%a%s%H$r5aak:]$K!"BeF~J8$N<B9T2s?t$^$G$r9MN8$KF~lilkh&$K!"BeF~J8$N<B9T2s?t$rCAG $N%(%C%8$N=E$_$H7FI=8=9k3HH7$?!#

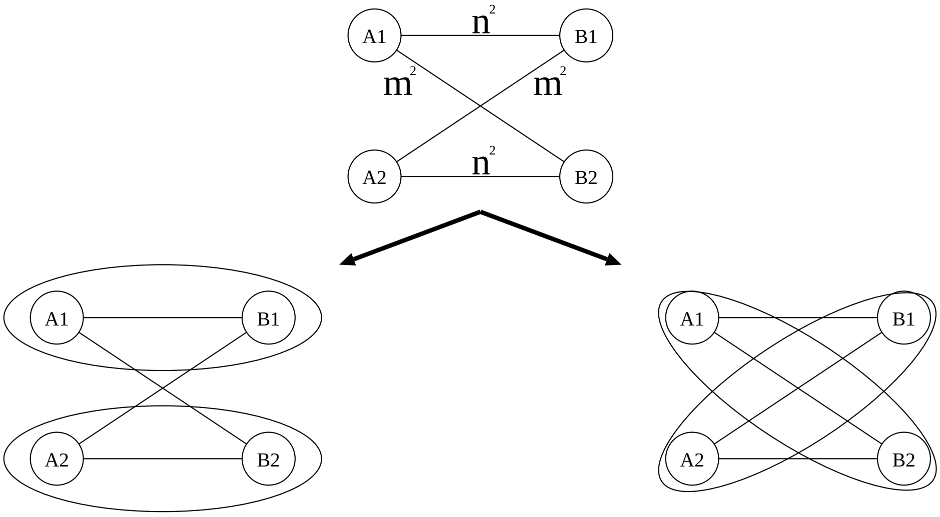
PARADIGM $O!"CAG $N%(%C%8$N=E$_$H7F!"$"$k2 DNG[NsJQ?t4V$N%(%C%8$K=>C?%"%i%$%s%a%s%H$r7?>l9gH=$l0J30$N%"%i%$%s%a%s%H$r7?H-N=l>lK$D$$$F@-G=8+@Qjr9T$$!"$=$N:9$r=E$_H7FD1k!#@-G=$r8+@Q$k$?$aKOB>$NJ,3d$N%Q%i%a!<%?$,I,MW$HJk$,!"$=li$N%Q%i%a!<%?$O!"IN<!85J}8~KbF1?t$N%W%m%;%C%5$r3djEvFil!"IN<!85$b%V%m%C%/J,3d$9kbNH7F07$&!#$3$NJ}K!$O!"@-G=8+@Qjr8F$S=P$9$?$a!"8uJd$N?t$,Hs>o$KB?$/Jk$H!"8+@Q$jN?aN%3%9%H$,Bg$-$/$JkH9M$($ilk!#$^$?!"%"%i%$%s%a%s%H0J30$N>pJs$rE,@Z$JbN$K7h$ail$F$$$J$$>l9g$O!"@-G=8+@Q$j$r@53N$K9TCFb=l[$I@53N$J>pJs$rF@$ilJ$$$3H,9M$($ilk!#

$=$NE@!"K\<jK!GOBeF~J8$N<B9T2s?t$r:,5r$K7h$akN$G%(%C%8$N=E$_7W;;$N%3%9%H$,7Z8:$5lk!#

<u>**CAG** $N:n@.%"%k%4%j%:%`</u> 0J>e$r$^$H$a$F!"CAG :n@.$N%"%k%4%j%:%`$r0J2<$K<($9!#

BeF~J8$N:8JU$,G[NsJQ?tG"kbNKBP7F0J2<$N=hM}$r9T$&!#

- :8JU$NG[NsJQ?t$KBP7F3F<!85$KBP1~$9$k%N!<%I$r:n@.9k
- 1&JU$N<0$K8=ol$kG[NsJQ?t$=l>lKBP7F!"0J2<$N=hM}$r9T$&!#
  - $=$NG[NsJQ?tKBP7$F3F<!85$KBP1~9k%N!<%I$r:n@.$9$k
  - :8JU$NG[NsJQ?t$KBP9k%N!<%I$H1&JU$NG[NsJQ?tKBP9$k%N!<%I$N9YFNAH$_9g$o$;$KBP7F$=$li,8_$$$K%"%U%#%s4X78K"k+I&$+$rD4Yk!#%"%U%#%s4X78G"$k>l9g$O!"$=$NBeF~J8$,2?2s<B9T$5lk$+$r7W;;$7!"$=lr=E$_$H7?%(%C%8$rM?$(il$?%N!<%I4V$K:n@.9k!#9GK3li$N%N!<%I4V$K%(%C%8$,:n@.$5lF$$$k>l9g$O!"$=$N%(%C%8$N=E$_$KBeF~J8$N<B9T2s?t$r2C$($k!#



?^3.8: CAG $N%N!<%I$N%0%k!<%WJ,$1

3.3.3 CAG $rMQ$$$?G[Ns<!85%"%i%$%s%a%s%H7hDj<jK!

M-0U$JG[Ns<!85%"%i%$%s%a%s%HHO!"CAG $N%N!<%I$r<!Nh$&$K%0%k!<%WJ,$1$9k3HG$"$k!#

> D $r%=!<%9%W%m%0%i%`$K8=ol$kG[NsJQ?t$N<!85?t$N:GBgCM$H7?H-!"9YFN%N!<%IrD$ 8D$NGSB>=89g$KJ,1k!#$?$@$7!"$=l>lN=89g$KB0$9k"$kl DNG[NsJQ?t$N<!85$KBP1~9k%N!<%I$O!"9b!9l DG$"$k!#

Nc$($P!"?^3.8 NNcG$O!"2 DNG[Ns<!85%"%i%$%s%a%s%H$,9M$($ilk!#

3liN2DG=$J%"%i%$%s%a%s%HNCf+$i!"DL?.NL$,:G>.HJkh$&$JbN$r5a$alP$h$$!#

$3$3$G!"CAG $N%(%C%8$N=E$_$O!"%"%U%#%s4X78$N=EMW@-$r<(9bNG$"$j!"$=$l$i2 DN<!85$rF1$8%0%k!<%WK9Y-G"kH$$$&>pJs$rI=8=$9$k!#$^$?!"CAG K*$$$F>e5-$NJ}K!$K=>CF%0%k!<%WJ,1r7?H-!"0[Jk%0%k!<%W4V$K8Y$,$k%(%C%8$O!"%a%C%;!<%8DL?.$NI,MW@-$rI=$7!"$=$N=E$_$,DL?.NL$rI=8=9kbNG"$k!#5U$KF1$8%0%k!<%WFb$KB8:_9k%(%C%8$O!"$=$N%"%i%$%s%a%s%H$,%(%C%8$N=E$_$NJ,$@$1G[NsJQ?t$N;2>H$N6I=j@-$rH?1G$7$?$bNKJC$F$$$k$H$$$&3H$rI=$9$3$HKJ$k!#

=>CF!":GE,$JG[Ns<!85%"%i%$%s%a%s%H$r5a$akH$$$&3H$O!"CAG K*$$$F!"%0%k!<%W4V$r7k$V%(%C%8$N=E$_$r:G>.$H9kh&$J%0%k!<%WJ,$1$r5a$akH$$$&3H$K5"Ce$5lk!#

K\8&5fGO!"A4FN2DG=$J%0%k!<%WJ,$1KBP7$FC5:w$r9T$&!#C5:w$N8uJd?t $O!"%W%m%0%i%`Cf$K8=$l$kG[NsJQ?t$N:GBg<!85?t$r$D$!"%W%m%0%i%`$K=P8=9k G[NsJQ?t$N8D?t$rm!"3FG[NsJQ?t$N<!85?t$r$n_i(i = 1, 2, \cdots, m)$!"$n_i = D$ HJ ki$ NCMN1$ Dr$i_{max}$$H9k$H!"0J2<$Nh&$KI=8=$G$-$k!#

$$\prod_{i=1, i \neq i_{max}}^{m} \frac{D!}{(D - n_i)!}$$

3NCM$O!"G[NsJQ?t$N?t(m) $NA}2C!"3FG[NsJQ?t$N<!85?t(n_i) $NA}2C$Khj GzH/E*$KA}2C$9$k!#$7$+$7!"<B:]$N%"%W%j%1!<%7%g%s$G$O=P8=$9$kG[NsJQ?t$N ?t(m) $K8B$j$,$"$j!"$^$?!"3FG[NsJQ?t$N<!85?t(n_i) $b3 <!85$^GNbN$,$[HsIG$"$k$?$a!"<B:]>e3NC5:w$r9T$CFb8=<BE*$J;~4V$G%"%i%$%s%a%s%H $r7hDj$9k3H,2DG=G"$k!#

3.4 %V%m%C%/%5%$%:Hf$N7hDj

K\%9%F%C%W$NL\E*$O!"JBNs<B9T;~$N%W%m%;%C%54V$NDL?.NL$r:GE,$K9kh& $JG[NsJQ?t4V$N3F<!85$NJ,3d$N%V%m%C%/%5%$%:$K$D$$$F$N4X78$r5aak3H$G $"$k!#BeF~J8$NG[NsJQ?t$N3F<!85$NE:;z<0$r2r@O9k3HKh$j!"2 DN<!85 4V$N%V%m%C%/%5%$%:KD$$$F$N@)Ls$r%V%m%C%/%5%$%:$NHfH7$F5a$a$k!#<B:] $N%V%m%C%/%5%$%:$N7hDj$O!"K\%9%F%C%W$N7hDj8e$K9Tol$k!#

2f!9$O!"$3$N%V%m%C%/%5%$%:Hf$N7hDj$K$"$?$j!"**BCG**(Block-Size Constraint Graph)[7] $H8F$Plk%0%i%U$rMxMQ$9$k!#0J2<!"BCG $,2?$rI=8=7?bNG" k+!"%a%C%;!<%8DL?.NL$r:GE,$K9k%V%m%C%/%5%$%:Hf$,BCG $rMQ$$$F$$$+ $K5a$ailk+KD$$$F=RYk!#

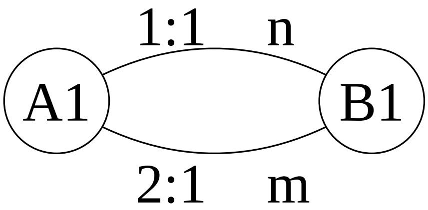
3.4.1 %W%m%0%i%`$+$iF@il$k%V%m%C%/%5%$%:Hf$N>pJs

$"$k%k!<%W%\%G%#Cf$NBeF~J8$G!": 8JU$NG[NsJQ?t$H1&JU$NG[NsJQ?t$N%"%U%#%s 4X78K"$k<!85$NE:;z<0$,$=l>l\alpha_1 * i + \beta_1$!"$\alpha_2 * i + \beta_2$$G$"$C$?$H$-!"%k!<%W JQ?ti NHj(k9YFNCMKBP7FN>T$,3d$jEvFilkC j>]%W%m%;%C%5$NMW AG$rEy7/9k$?$aKO!"2 DN<!85$NJ,3d$N%V%m%C%/%5%$%:$NHf$,$|\alpha_1| : |\alpha_2|$ HJk3H,I,MWG"$k!#

K\%9%F%C%WGO!"3N>r7o$KCeL\$7!"%W%m%0%i%`A4BNH7F3li$N>r7o$r 2DG=$J8B$jK~$?$9h&$J%V%m%C%/%5%$%:Hf$r5a$ak3HrL\E*H9$k!#

7+$7!"%W%m%0%i%`A4BN$H7F!"L7=bJ/3Nh&$J%V%m%C%/%5%$%:Hf$r5a$a il$J$$>l9gb"$k!#$9Jo$A!"%V%m%C%/%5%$%:Hf$K6%9g$,@8$8$k>l9g$,$"$k!#

```
DO i = 0 , n - 1
  A(i) = 𝓕(B(i))
ENDDO
     ⋮
DO i = 0 , m - 1
  A(2*i) = 𝓕(B(i))
ENDDO
```



?^3.10: ?^3.9 KBP9$kBCG

?^3.9: %V%m%C%/%5%$%:Hf$K6%9g$r5/$3 $9%W%m%0%i%`Nc1

Nc$($P!"?^3.9 Nh$&$J%W%m%0%i%`NNcG$O!"1 $DL\$N%k!<%W%\%G%#$NBeF~J8$+iO!"A_1 :B_1 $= 1 : 1$ $H$$$&%V%m%C%/%5%$%:Hf$,5a$^$k$,!"2 $DL\$N%k!<%W%\%G%#$NBeF~J8$+iO!"A_1 :B_1 $= 2 : 1$ $H$$$&%V%m%C%/%5%$%:Hf$,5a$^$k!#3NN>J}$rK~$?9h$&$JHf$OB8:_$7$J$$!#

$^$?!"?^3.11 Nh$&$J%W%m%0%i%`NNcG$O!"1 $DL\$N%k!<%W%\%G%#$NBeF~J8$+iO!"A_1 :B_1 $= 1 : 1$!"A_2 :B_2 $= 1 : 1$ $H$$$&%V%m%C%/%5%$%:Hf$,5a$^$j!"2 $DL\$N%k!<%W%\%G%#$NBeF~J8$+iO!"A_1 :B_2 $= 1 : 1$!"A_2 :B_1 $= 1 : 2$ $H$$$&%V%m%C%/%5%$%:Hf$,5a$^$k!#3N>l9g$b!"$9YF$rK~$?9h$&$J%V%m%C%/%5%$%:Hf$OB8:_7J$$!#

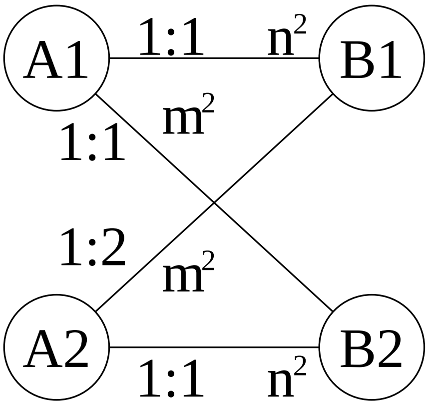
3Nh&$K%V%m%C%/%5%$%:Hf$N>r7o$K6%9g$,$"$k>l9g!"6%9g$9$k>r7o$N$&$A!">r7o$rK~$?$;$J$$$?aK5/3k@-G=Dc2<$,:G>.$G$"$kh&$J>r7o$r5a$a!"$=$N>r7o$r<h$j=|$/I,MW$,$"$k!#$3$NLdBj$r!"BCG(Blocksize Constraint Graph) $H8F$PlkL58~%0%i%U$rMQ$$$F2r$/!#

3.4.2 BCG $,;}$D>pJs

BCG $N%N!<%I$OG[NsJQ?t$N<!855$rI=$9!#%(%C%8$O$=$NN>C<$N%N!<%I$KBP1~9kG[NsJQ?t$N<!854V$K%V%m%C%/%5%$%:Hf$N@)Ls$,$"k3HrI=$7!"%(%C%8$NB0@-H7$F%V%m%C%/%5%$%:Hf$rJ];}$9$k!#$5iK!"K\<jK!GO%(%C%8$NB0@-$K!"$=$N%(%C%8$,J];}$9$k%V%m%C%/%5%$%:Hf$rF3$$$?3FBeF~J8$N<B9T2s?t$NOB$rIUM?$9$k!#$3$NM}M3$OKD$$$F$O!"<!@a$N!VBCG Kh$k%V%m%C%/%5%$%:Hf$N7hDj<jK!!W$G=RYk!#

Nc$($P!"?^3.9!"?^3.11 $N%W%m%0%i%`$KBP9kBCG O=l>$l?^3.10!"?^

```
DO j = 0 , n - 1
  DO i = 0 , n - 1
    A(i,j) = $\mathcal{F}$(B(i,j))
  ENDDO
ENDDO
    ⋮
DO j = 0 , m - 1
  DO i = 0 , m - 1
    A(i,j) = $\mathcal{F}$(B(2*j,i))
  ENDDO
ENDDO
```



?^3.12: ?^3.11 KBP9$kBCG

?^3.11: %V%m%C%/%5%$%:Hf$K6%9g$r5/$3
$9%W%m%0%i%`Nc2

3.12 Nh$&$KJk!#

BCG $N%N!<%I$O!"CAG $HF1MM$KG[NsJQ?t$N<!85$rI=8=9kbNG"$j!"$^$?!"%(%C%8$NM-L5$NH=Dj$bCAG $N:n@.$HF1MM!"G[NsJQ?tI&7NBeF~J8$r2r@O$7!"G[Ns<!854V$N%"%U%#%s4X78$rD4Yk3H$G9T$&$?$a!"<B:]KO!"CAG $N:n@.$HF1;~$KBCG $N:n@.$b9T$&!#=>$C$F!"$3$N%9%F%C%W$G$O!"$9GK:n@.5l$?BCG rbHK%V%m%C%/%5%$%:Hf$r5aak3HKJ$k!#

3.4.3 BCG Kh$k%V%m%C%/%5%$%:Hf$N7hDj<jK!

$"$k%W%m%0%i%`KBP7$F%V%m%C%/%5%$%:Hf$K6%9g$,$"$kNO!"BCG $K%5%$%/%k$,B8:_$9kH$-$K8Bil$k!#$J$<$JiP!"BCG $K%5%$%/%k$,B8:_$7$J$$>l9g$O!"<!$N<j=gKhCF!"%V%m%C%/%5%$%:Hf$r0l0U$K5a$ak3H,2DG=$@$+iG$"$k!#

1. 9YFN%N!<%IKBP7$F0J2<$r9T$&

    8=:_=hM}BP>]G"$k%N!<%I$r v H9$k!#$v$ $H!"$v$ $K@\B3$9$k%N!<%I $u_i, (i = 1, 2, \cdots \mathrm{degree}_v)$ $N%V%m%C%/%5%$%:Hf$,$\alpha_i : \beta_i (i = 1, 2, \cdots \mathrm{degree}_v)$ NH$-!"$\alpha_1, \alpha_2, \cdots, \alpha_{\mathrm{degree}_v}$ $N:G>.8xG\?t$lcm_v$ $r5a$a!"v Hu_i$ $N%V%m%C%/%5%$%:Hfr\alpha_i : \beta_i$ $+$i $lcm_v : (lcm_v / \alpha_i) * \beta_i$ $K99?7$9$k

2. 9YFN%N!<%IKBP7!"$\alpha_1 = \alpha_2 = \cdots = \alpha_{\mathrm{degree}_v}$ G"lP=*N;!#$=$&GJ

$$>19g$O1.$r7+jJV9!#

=>CF!"%V%m%C%/%5%$%:Hf$N6%9g$r2r7h$9k?aK$O!"BCG $+$i$$$/D+$N%(%C%8$r=|5n$7%5%$%/%k$rL5$/$;$PNI$$!#

7+$7!"%5%$%/%k$N=|5n$N$?$a$K%(%C%8$,:o=|$5$1$?>19g!"$=$N%(%C%8$,;}$D>pJs$O!"K\%9%F%C%W$GF@$i1k%V%m%C%/%5%$%:Hf$KH?1G$5$1$J$$!#=>CF!"G-$k8B$j=EMW@-$ND c$$%(%C%8$@$1$r=|5n$9kI,MW,$"$k!#3N=EMW@-$rH=CG$9k?aK!"%(%C%8KBP7$F=E$_$rB0@-$H7FD1$k!#K\<jK!$G$O=E$_H7$F!"$=$N%(%C%8$,F@i1kbHHJC$?BeF~J8$,<B9T$5$1$k2s?t$r:NMQ9k!#$=$NM}M3$O!"<B9T2s?t$,hjB?$$BeF~J8KBP7$FDL?.NL$r:GE,H9kh$&$J%V%m%C%/%5%$%:Hf$r:NMQ7?J}$,!"<B9T2s?t$,>/$J$$BeF~J8KBP7$FDL?.NL$r:GE,H9kh$&$J%V%m%C%/%5%$%:Hf$r:NMQ9khj$bDL?.NL$rDc8:G-k+iG$"$k!#

0J>eN3H+$i!":GE,$J%V%m%C%/%5%$%:Hf$N7hDj$O!"BCG K*$$$F%5%$%/%k$,B8:_7J$$$h$&$J%(%C%8=89gN&$A!"$=$N=89g$KB0$9$k%(%C%8$N=E$_NOB,:GBgHJkh$&$J=89g$r5a$a$kLdBj$HJk!#

K\8&5fGO!"<!$N<j=g$KhC$F!"6%9g$r2r7h9k$?$a$K=|5n$9Y-%(%C%8$N=89g$r5aak!#

1. BCG N9YF$N%(%C%8$KBP7F!"0J2<$r9T$&!#
   8=:_=hM}Cf$N%(%C%8$NN>C<$N%N!<%I$r u, v H9$k!# u, v 4V$KB8:_$9k9YF$N%(%C%8$N=89g$+$i:GBg$N=E$_$r;}$D%(%C%8$r1 $DA*$S!"$=$10J30$N%(%C%8$r9Y$F:o=|$9$k!#
2. BCG $+$i%5%$%/%k$r8!=P9k!#%5%$%/%k$,L5$1$1$P=*N;!#8!=P$5$1$?%5%$%/%k$r9=@.9k%(%C%8=89gN&$A!":G>.$N=E$_$r;}$D%(%C%8$r1 $DA*$S!"$=$N%(%C%8$r:o=|$9$k!#2.$r7+jJV9!#

3.5 %V%m%C%/%5%$%:$N7hDj

K\%9%F%C%WGO!"%W%m%;%C%54V$NI i2Y%P%i%s%9!"DL?.NL$r:GE,K9kh$&$J%V%m%C%/%5%$%:$N7hDj$r9T$&3H$rL\E*$H9k!#:#!"A0%9%F%C%W$G!"5a$^C?$"$kG[Ns<!85$N%0%k!<%WFb$N3FG[Ns<!85$N%V%m%C%/%5%$%:Hfr\alpha_1 : \alpha_2 : \cdots : \alpha_m$$H9k!#3N;~!"%V%m%C%/%5%$%:$N7hDjHO!"$"$k@0Dj?t$b_k$$rCj>]%W%m%;%C%5$NBh k <!85L\$KBP1~E1i1$?G[Ns<!85$N%0%k!<%WKBP7$F5a$a!"$=$N%0%k!<%W$KB0$9$kG[Ns<!85$N%V%m%C%/%5%$%:$r$=$1$>$1$(b_k \times \alpha_1), (b_k \times \alpha_2), \cdots, (b_k \times \alpha_m)$$H7F5aak3HG"$k!#

```
DO j = 1 , n
  DO i = j , n
    A(i,j)=$\mathcal{F}$(A(i,j),B(i,j))
  ENDDO
ENDDO
```

?^3.13: Ii2Y%P%i%s%9$r9MN8$9Y-%W%m%0%i%`Nc

```
DO i = 1 , n
  A(i)=$\mathcal{F}$(A(i-1),A(i+1))
ENDDO
```

?^3.14: NY@\MWAG$N%"%/%;%9$,$"$k%W%m%0%i%`Nc

3.5.1 %V%m%C%/%5%$%:$,@-G=$KM?$($k1F6A

@0Dj?t$b_k$$rDjak$?$a$K!"$b_k$$NCM$NJQ2=$,@-G=$KM?$($k1F6A$K$D$$$F9M;!9k!#Ii2Y%P%i%s%9XN1F6A $"$k%k!<%W3+;OCM!"=*N;CM$K!"$=$N30B&$N%k!<%WJQ?t$,4^$^lk>l9g!"$=$N%k!<%W$N<B9T2s?t$O!"30B&%k!<%W$N%k!<%WJQ?t$NCMKhCFJQ2=9k!#3Nh&$J%k!<%W$N%k!<%WJQ?t$rE:;z<0$K4^$`G[NsJQ?t$,%k!<%W%\%G%#$NBeF~J8$N:8JU$K8=$o$l$k>l9g!"$3$NG[NsJQ?t$NJ,3d$N%V%m%C%/%5%$%:$,Bg$-$$$H!"3F%W%m%;%C%5$K3d$jEv$F$i$l$k=hM}NL$KIT6Q9U$,@8$8$k!#5U$K!"%V%m%C%/%5%$%:$r>.$5$/$7$?J}$,Ii2Y%P%i%s%9$rNI$/$G$-$k!#Nc$($P!"?^3.13 $N%W%m%0%i%`$NNcGO!"FbB&$Ni %k!<%W$N3+;OCM$,!"30B&$N%k!<%WJQ?tj G"$k!#$3N?$a!"BeF~J8$N:8JUK"$kG[Ns<!85$A_1$!"$A_2$$H$b!"%V%m%C%/%5%$%:$r2DG=$J8B$j>.$5$/$7$?J}$,Ii2Y%P%i%s%9$,NI$/Jj!"<B9T;~4V$rC;=L$G$-$k!#

%W%m%;%C%54V$NDL?.NL$X$N1F6A 0lJ}!"NY@\MWAG$N%"%/%;%9$,B?$$%W%m%0%i%`KBP7FO!"%V%m%C%/%5%$%:$rBg$-$/9kJ}$,DL?.NL$rDc8:G-$k!#$J$<$JiP!"%V%m%C%/%5%$%:$,>.$5$$$H!"%W%m%;%C%56-3&$,A}$(!"A4%G!<%?KBP7$FNY@\MWAG$I$&$7$,0[$J$k%W%m%;%C%5$KG[CV5l$k3d9g$,9b$/$J$j!"$=$N7k2L!"DL?.$,IQH/9k$+$iG"$k!#5U$K!"%V%m%C%/%5%$%:$,Bg$-$$>l9g$O!"NY@\MWAG$I$&$7$,F1$8%W%m%;%C%5$KG[CV$5lk3d9g$,9b$$$?$a!"DL?.NL$rDc8:G-$k!#Nc$($P!"?^3.14 $N%W%m%0%i%`$NNcGO!"7W;;N?aKG[NsJQ?tA $NNY@\MWAG$rF1;~KI,MWH9k!#3N$?$a!"G[Ns<!85$A_1$$O!"%V%m%C%/%5%$%:$r2DG=$J8B$jBg$-$/7?J}$,DL?.NL$rDc8:G-$k!#

3.5.2 %V%m%C%/%5%$%:$N7hDj

3Nh&$K!"%V%m%C%/%5%$%:$O!"%W%m%;%C%54V$NIi2Y%P%i%s%9HI,MWHJkDL?.NL$r8_$$$K5U$NJ}8~$K@-G=$rJQ2=$5$;$k%Q%i%a!<%?G"k?$a!"N>J}$r0l

85E*$KH=CG$9$k<jCJ$,I,MWHJCF$/$k!#

3NLdBjKBP7$F!"K\<jK!$G$O8=:_$N=j!"%k!<%W$N<B9T2s?t$,3OB&%k!<%W$N%k!<%WJQ?t$Khi$:0lDj$G$"$k$+$I$&$+$K4X$oi:!"9YFN<!85$rA0%9%F%C%W$G5a$^$C$?%V%m%C%/%5%$%:Hf$rK~$?$9HO0O$G2DG=$J8B$jBg$-$J%V%m%C%/%5%$%:$GJ,3d9kh&K7$F$$$k!#

3liN2DG=$J2~NI$H7F$O!"%k!<%W$N<B9T2s?t$,3OB&%k!<%W$NJQ?tKhi:0lDjHJiJ$$>l9g$O!"%V%m%C%/%5%$%:$rMM!9$J@0Dj?t$b_k$$H7F8uJd$r5s$2!"3liKBP7F@-G=8+@Qjr7F%V%m%C%/%5%$%:$r7hDj9k$H$$$&J}K!$,9M$(il$k!#

$3$3$G!"!ZDG=$J8BjBg-$J%V%m%C%/%5%$%:!WHO!"<!$N<0$KhC$F5a$ailkb$ KhCFDjailk%V%m%C%/%5%$%:$G$"$k!#

$$b = \max_{1 \leq i \leq m} \left\{ \left\lceil \frac{N_i}{\alpha_i \times P} \right\rceil \right\}$$

$?$@$7!"$N_1, N_2, \cdots, N_m$ $O!"G[Ns<!85%"%i%$%s%a%s%HKhCF5aail?%0%k!<%WJ,1N!"$"$k%0%k!<%WFb$N3FG[Ns<!85$NG[NsMWAG?t$rI=$7!"P $OG[Ns<!85$N%0%k!<%W$K3d$jEvFilk<B%W%m%;%C%5?t$rI=$9!#

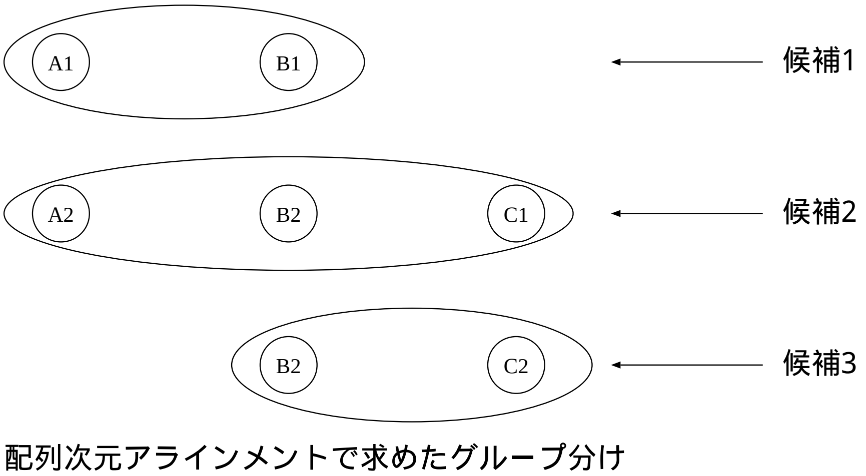
3.6 J,3d<!85$N7hDj

K\%9%F%C%WGO!"JBNs@-$r:G$b0z$-=P$;kh$&$JG[NsJQ?t$NJ,3d<!85$r5aak3H$rL\E*$H9k!#9G$K!"$3l^GN%9%F%C%W$GG[NsJQ?t$NJ,3d$K4X$9$kAjBPE*$JG[CV4X78$,Cj>]%W%m%;%C%5$X$NG[NsMWAG$N3djEvFH7$FI=8=$5lF$$$k!#K\%9%F%C%WGO!"Cj>]%W%m%;%C%5$N<!85$NCf$+$i!"J,3d9k<!85$r7hDj$9$k!#J,3d$5lkh&7hDj5l$?Cj>]%W%m%;%C%5$N<!85$KBP1~$E1il?G[Ns<!85O9YFJ,3d5l$k!#$3$N:]!"J,3d$N%V%m%C%/%5%$%:$O!"A0%9%F%C%W$G5a$^C?CM$r;HMQ$9$k!#

$?$@$7!"K\<jK!$G$O8=:_$N=j!"J,3d<!85$,$1$ <!85$N>l9g$N$_$rC5:w$NBP>]$H7F$$$k!#

?^3.15 NNcG$O!"$=l>l2$!"$3$!"$2$ <!85N3$ DNG[NsJQ?tA!"B!"C $N3F<!85$,!"G[Ns<!85%"%i%$%s%a%s%H$N%9%F%C%WKhCF%0%k!<%WJ,$1$5lF$$$k!#K\%9%F%C%WGO!"3liN3F%0%k!<%WNCf+i1$ DN%0%k!<%W$rA*Br$9k3HKhC$FJ,3d<!85$r5aak!#

J,3d<!85$N7hDj$G$O!"G[Ns<!85$N3F%0%k!<%W$rJ,3d$7$?>l9g$N@-G=8+@Qjr



?^3.15: J,3d<!8 5$N7hDj

9T$$!":GE,$J<B9T;~4V$HJk%0%k!<%W$rJ,3d$9k3HH9k!#6qBNE*$J8+@Q$j J}K!$OBh4 >O$G>\$7$/=R$Y$k!#

4 J,3d<!8 57hDjN?aN@-G=8+@QJ}K!

K\>OGO!"G[Ns%G!<%?$NJ,3d<!8 5$r5aak$?$a$K!"M?$(il$?%G!<%?J,3d$K hC$F%k!<%W$,INh&$K<B9T$5lkN+$rB*$(!"$=$N<B9T;~4V$r8+@Q$kJ}K! $r=R$Y$k!#$3$N8+@Q$O!"%G!<%?J,3d$NM%Nt$rH=Dj9k3H$rBh0l$NL\E*H7!" I,$:$7$b@dBPE*$J<B9T;~4V$^$G@5$7$/5aakI,MWOJ$$!#

K\8+@QJ}K!GO!"E,MQ2DG=$J%W%m%0%i%`$NHO0O$K@)8B$,$"$k$?$a!"$3$NHO0O 30$N%W%m%0%i%`$KBP9kJ,3d<!8 57hDj<jK!$,I,MW$KJCF/$k!#$=3G!"8+@Q j,E,MQG-$J$$%W%m%0%i%`KBP9$kJ,3d<!8 5$N7hDj<jK!H7$F!"G[Ns<!8 5J, 3dE,@-EY$rMQ$$$?J}K!KD$$$F$bK\>O$G=RYk!#

$^$:!"Bh4.1 @a!"Bh4.2 @a$GK\<jK!$G$N8+@QJ}K!$N35MW$HE,MQ2DG=HO0O$r=R$Y $k!#Bh4.3 @a$GM?$($il?%W%m%0%i%`$HJ,3d<!8 5$N8uJd$+$iK\>O$G=R$Y$k8+@Q J}K!$,E,MQ2DG=$+$I$&$+$rH=CG$9$kJ}K!$K$D$$$F=RYk!#5i$K!"Bh4.4 @a$H 4.5 @a$G6qBNE*$J@-G=8+@QJ}K!KD$$$F=R$Y!":G8e$KBh4.6 @a$GK\8+@Q$jJ}K!$, E,MQ$G$-$J$+$C$?>l9g$KE,MQ5l$k!"G[Ns<!8 5J,3dE,@-EY$rMQ$$$?J,3d<!8 5$N 7hDj<jK!$K$D$$$F=RYk!#

4.1 8+@QJ}K!$N35MW

%k!<%W$rJBNs2=$7$?$H$-!"$=$N<B9T;~4V$O!"0J2<$N3F;~4V$KhC$F@j$ail$k!#

- 3F%W%m%;%C%5GN1i;;;~4V

  3F%W%m%;%C%5$,<B9T$9$k1i;;$K$+$+$k;~4V

- 3F%W%m%;%C%5NBTA;~4V

  %G!<%?0MB8N?$a!"B>$N%W%m%;%C%5$+$iDL?.KhCF%G!<%?$,Aw?.$5lF$/$k$^$G<B9T$rCfCG$5lk;~4V

- %W%m%;%C%54V$NDL?.;~4V

  <B9TN?aK;2>H5l$k%G!<%?$rB>$N%W%m%;%C%5$,;}$D>l9g$KI,MWHJ$k%G!<%?<u?.%3%9%H!#$^$?!"B>$N%W%m%;%C%5GN<B9T$G;2>H$5lk%G!<%?$r;}$D>l9gKI,MWHJk%G!<%?Aw?.%3%9%H!#

K\<jK!GO>e5-$N3F;~4V$r!"<!Nh$&$K8+@Q$k!#

$^$:%?%9%/!"4pK\%?%9%/=89g$H$$$&35G0$rF3F~$7!"3liN8D?tKhCF3F%W%m%;%C%5NBTA;~4V$^$G4^a?3F%W%m%;%C%5GN=hM};~4V$r!"DL?.$N%3%9%H$,0$G$"$kH7$F8+@Q$k!#3l$O!"%?%9%/4V$N0MB84X78rbHK8+@Qjr9T$&!#

<!$K3F%W%m%;%C%54V$GH/@8$9$kDL?.$N%3%9%H$r8+@Qk3HKhj!":G=*E*$J8+@Q$j;~4V$rF@$k!#

>e5-$N3FI>2A9`L\$r8+@Qk?aK$O!"%?%9%/$N%W%m%;%C%5XN3djEvFJ}$d!"DL?.$N%9%1%8%e!<%j%s%0K!JI$N>pJs$,I,MWG"k,!"3liOJBNs2=%3%s%Q%$%i$KhC$F0[$J$k!#6qBNE*$J0c$$$d!"8+@QK*$$$FGD0.9kI,MWN"$kJBNs2=%3%s%Q%$%i$NFCD'$N07$$$K$D$$$F$OBh4.4@a$G=RYk!#

4.2 E,MQ2DG=$JHO0O

$3$3$G!"%W%m%;%C%5$X3djEvF$k=hM}$N:G>.C10LH7$F!"%?%9%/$H$$$&35G0$rF3F~$9$k!#%W%m%0%i%`A4BN$r%?%9%/$N=89g$H$7$F07$&$3$H$G!"A4%W%m%;%C%5$G$N=hM}$NN.$l$rBg6IE*$KB*$($d$9$/$J$k$3$H$,4|BT$G$-$k!#K\<jK!$G$O!"%k!<%W$N<B9T>u67$rB*$($k$3$H$rL\E*$H$9$k$?$a!"%k!<%W$N1%$%?%l!<%7%g%s$r1$D$N%?%9%/$H$7$F07$&!#$3$N$?$a$K$O!"JBNs2=$5$l$k%W%m%0%i%`$N%G!<%?J,3d$K<!$Nh&$J@)8B$,I,MWG"$k!#

$"$k%$%?%l!<%7%g%sFb$NBeF~J8$N:8JU$H7F%"%/%;%9$5$l$kG[Ns%G!<%?$N=jM-<TO9YFF101%W%m%;%C%5G"$k

Owner Computes Rule $K=>$G$?JBNs2=$r9T$&$?$a!"$3$N>r7o$rK~$?$7$F$$$l$P!"%$%?%l!<%7%g%sFb$N<B9TJ8$O9Y$FF1$8%W%m%;%C%5$G<B9T$5lk3H$,J]>Z$5lk!#

$^$?K\<jK!GO!"@-G=8+@Qjr@53N$K9T$&$?$a$K!"8+@Q$j2DG=$J%W%m%0%i%`$KBP7F0J2<$N@)8B$r@_1k3HH7$?!#

1. %$%?%l!<%7%g%s$K$^$?$,$k0MB8%Y%/%H%k$O!"%$%?%l!<%7%g%sKhi:0lDjG"$k!#$3Nh$&$J0MB8$runiform $J0MB8$H8F$V!#
2. 3F%k!<%W$NH?I|2s?t$O!"3 0B&%k!<%W$N%k!<%WJQ?t$NCMKhi:0lDjG+$D4{CN!#$9Jo$A!"%k!<%WJ8$N3+;OCM!"=*N;CM!"%9%F%C%WItJ,$,!"$9YF%3%s%Q%$%k;~$K2r@O2DG=$J@0Dj?t$G$"$j!"$=$1iK$O3 0B&%k!<%W$N%k!<%WJQ?t$,4^$^1J$$!#
3. %k!<%W%\%G%#Fb$K>r7oJ,4t$,J/!"=hM}$N?J9T$O!"F~NO%G!<%?Khi:0l0U$K7h$^$k!#
4. %Q!<%U%'%/%H%k!<%WG"$k!#%Q!<%U%'%/%H%k!<%W$H$O!"<B9TJ8$,%k!<%W%M%9%H$N:GFb%k!<%W$N%\%G%#K7$+8=$1$J$$$h$&$J%k!<%W$r;X$9!#

$^$?!"$3$1iN@)8B$rK~$?7F$$$k%W%m%0%i%`KBP7FO!"0J2<N3H,2>DjG-$k!#

- (1. $+$i) $"$kF10lG[NsMWAG$NDj5A!";2>H$N$?$a$KH/@8$9$kDL?.$O!"0MB85wN%$,>o$K0lDjG"k3H+$i!"$=$NDL?.Aj<j$OFCDj$N%W%m%;%C%5$@1K8Bi1$k!#
- (2. $+$i) %k!<%W$NA4=hM}NL$O!"%3%s%Q%$%k;~$KJ,$+$k!#
- (3. $+$i) 3F%$%?%l!<%7%g%s!"$9Jo$A3F%?%9%/$N<B9T;~4V$O0lDj$G$"$k!#
- (2.4. $+$i) %k!<%W$NA4=hM}NL$O!"%?%9%/$r4pK\C10L$H7F!"$=$N?t$GI=8=$G$-$k!#

4.3 @-G=8+@QE,MQ$N2DH]$NH=CG

Bh4.2 @a$G=R$Y$?E,MQ2DG=HO0O$O!"J,3d<!85$N8uJd$KBP9k>r7o$H!"%W%m%0%i%`$=$N$b$N$,;}$D@-<A$KBP$9$k>r7o$,4^$^$1$k!#$3$N$?$a!"1$D$NC`<!%=!<%9%W%m%0%i%`KBP7$F!"$"$kJ,3d$KBP7F$O8+@Q$j$,2DG=$G$"$k$,!"B>$NJ,3dKBP7FO8+@Q$,E,MQ$G$-$J$$!"$H$$$&3H$,5/$3j($k!#$3$1$i$NJ,3d$NAH$_9g$o$;$,J,3d8uJdH7$F4^$^1F$$$k>l9g!"8+@Q2DG=JbNK4X7FN_8+@Qjr9TGF$b!"J,3d<!85$r7hDj9k$?$aK=1i$rHf3S$,G-$J$$$?$aL50UL#$G$"$k!#

7?$,$G$F!"E,MQ$N2DH]$NH=CG$O!"J,3d<!85$N8uJd$N9YFKBP7FBh4.2 $N>r7o$rK~$?$9$+$I$&$+$rD4$Y!"9YF,>r7o$rK~$?$;$P3F8uJdKBP7$F8+@Q$r

```
DO l_1 = ...   (s_1H?I|)
  :
  DO l_i = ...   (s_iH?I|)
   ↑JBNs2=$5$1$k%k!<%W
    :
    DO l_m = ...   (s_mH?I|)
      {%k!<%W%\%G%#$N<B9TJ8}
    ENDDO
    :
  ENDDO
  :
ENDDO
```

C`<!%W%m%0%i%`

```
DO l_1 = ...   (s_1H?I|)
  :
  DO l_i ∈ { <+%W%m%;%C%5$,<B9T$9
             $k%$%s%G%C%/%9=89g }
    :
    DO l_m = ...   (s_mH?I|)
      task(l_1, ···, l_i, ···, l_m) $N<B9T
    ENDDO
    :
  ENDDO
  :
ENDDO
```

JBNs2=%W%m%0%i%`

?^4.1: %k!<%W$NJBNs2=

E,MQ9k!#$=$&GJ$$>l9g$O!"$9YFG[Ns<!85J,3dE,@-EYKh$kJ}K!$GJ,3d<!85$r5a$a$k!#

4.4 JBNs2=%3%s%Q%$%i$KhkJBNs2=$*$h$S:GE,2=

K\@aGO!"Bh4.2 @a$G=R$Y$?%k!<%W$KBP7F9Tol$k!"JBNs2=%3%s%Q%$%iKh$kJBNs2=!":GE,2=$N$&$A!"8+@QjK$*$$$FGD0.9kI,MWN"$kFCD'$K$D$$$F=R$Y!"@-G=8+@Q$jK*$$$F!"3liNFCD'rI$&07$&$+$K$D$$$F=RYk!#

4.4.1 %?%9%/$N3d$jEv$F

JBNs2=%3%s%Q%$%i$,Owner Computes Rule $K=>$C$?=hM}$NJ,3d$r9T$&$?$a!"G[NsJQ?t$NJ,3d<!85$K8=lk%k!<%WJQ?t$r%$%s%G%C%/%9H9$k%k!<%W$,JBNs2=5l$k!#3F%$%?%l!<%7%g%s$O!"A0@a$G=RY?h&$K%?%9%/$KBP1~$7!"3F%k!<%W%M%9%H$N%$%s%G%C%/%9CM$KhC$F<1JL$5lk!#JBNs2=5l$?%k!<%W$O!"<+%W%m%;%C%5$NC4EvJ,$N%k!<%W%$%s%G%C%/%9$KBP1~9k%?%9%/$N<B9T$@1r9T$&!#?^4.1 $O!"m =E%k!<%W$NBhi HVL\$N%k!<%W%M%9%H$,JBNs2=$5lF$$$k3H$rI=$9!#

3N?^$N!"!V<+%W%m%;%C%5$,<B9T9k%$%s%G%C%/%9=89g!W$O!"%k!<%W%\%G%#$NBeF~J8$N:8JU$NG[NsJQ?t$NJ,3dKhCF5aailk!#%V%m%C%/%5%$%/%j%C%/J,3d$r7F$$$k>l9g!"3liN%$%s%G%C%/%9=89g$NHO0O$O!"J,3d$N5,B'@-$+$i!"%k!<%W$r%9%H%j%C%W%^%$%K%s%0$9$k3HKhCFI=8=G-$k[1]!#

@-G=8+@QjKI,MW$J>pJs$O!"3liN%9%H%j%C%W%^%$%K%s%0$KhC$F@8@.$5l?FbB&!"30B&%k!<%W$,$=l>$l2?2s<B9T$5lk$+$H$$$&3HG"$k!#$3li

[illegible]

[illegible] l_i [illegible] $BlockSize$ [illegible] $a \times l_i + b$ [illegible] P [illegible] l_i [illegible] s_i [illegible] $(outer\_l_i)$ [illegible] n [illegible] $(inner\_l_i)$ [illegible] B [illegible] n [illegible] B [illegible]

$$B = \frac{BlockSize}{a} \tag{4.1}$$

$$n = \left\lceil \frac{s_i}{Stride} \right\rceil, Stride = B \times P \tag{4.2}$$

[illegible] (4.1) [illegible] $BlockSize$ [illegible] a [illegible] B [illegible] (4.1) [illegible] $BlockSize$ [illegible] B [illegible]

[illegible] 4.2 [illegible]

[illegible]

4.4.2 [illegible]

[illegible]

[illegible]

[illegible]

```
⋮
DO $outer\_l_i$ =...   ( %9%F%C%W= $Stride$!"$n$ H?I|)
    DO $inner\_l_i$ =...   ( %9%F%C%W= 1!"$B$ H?I| )
      ⋮
        DO $l_m$= ...
          call task($l_1, \cdots, l_i, \cdots, l_m$)        ←4pK\%?%9%/=89g
        ENDDO
      ⋮
    ENDDO
ENDDO
⋮
```

?^4.2: %9%H%j%C%W%^%$%K%s%0$5l?JBNs2=%3!<%I

7F$+$iAw?.9k$+!"Dj5A$;$:$KAw?.G-k+KhCFJ,N`$G$-$k!#%G!<%?$NAw?.$KBP$7$F!">e5-$NJ,N`$=$l$>$l$NAw?.$KBP7F9Tol$k0lHIE*$J:GE,2=$H!"8+@Q$jK*$$$F$=$liN:GE,2=rI$&<h$j07$&$+KD$$$F0J2<$K=R$Y$k!#

<+%W%m%;%C%5$,Dj5A$7$?%G!<%?$NAw?. %a%C%;!<%88r497?$NJBNs7W;;5!$NDL?.$N5/F0%3%9%H$O0lHIE*$KHs>o$K9b$$$?$a!"2DG=$J8B$j5/F02s?t$r8:i9I,MW$,$"$k!#$3N?$a!"<+%W%m%;%C%5$,Dj5A7?%G!<%?$,F1$8%W%m%;%C%5$XAw?.$5lk>l9g!"3liN%G!<%?$r1 DN%a%C%;!<%8H7$F%Q%C%-%s%0$7!"0lEYKAwk$H$$$&:GE,2=(%a%C%;!<%8$N%Y%/%H%k2=)$,E,MQ5l$k!#0lJ}!"%G!<%?$r<u?.9kB&$O!"$=$N%G!<%?$r<u1Hk^$G<B9T$,?JailJ$$!#7?$,$C$F!"Aw?.B&$G7W;;5l$?%G!<%?$O2DG=$J8B$jAa$/Aw?.$9kI,MW,$"$k!#H3m,%a%C%;!<%8$N%Y%/%H%k2=$,5lkH!"%Y%/%H%k2=9Y$-%G!<%?$,9Y$FB7$&$^$GAw?.$,CY$i5lk?$a!"$=$NJ,<u?.%W%m%;%C%5$OBT$?$5lk3HKJ$k!#$^$?!"%Y%/%H%k2=$r7J$$$G!"%G!<%?$,7W;;$5$l<!BhAw?.$9k3HK9k$H!"DL?.$N5/F02s?t$,A}$(kH$$$&7k2LKJ$k!#$3li$N%H%l!<%I%*%U$N4X78$O!"%Y%/%H%kD9$H7F%Q%i%a!<%?2=9k3H$,$G$-$k!#K\8&5fGO3N%Y%/%H%kD9$O4{CN$NbNG"kH9k!#

JBNs2=%3%s%Q%$%iTINPAR$G$O!"K\<jK!$G07$&%W%m%0%i%`$KBP7F$O!"4pK\%?%9%/=89g$rC10LH7$F%Y%/%H%k2=$r9T$&!#$=3G!"K\8&5fGO!"<+%W%m%;%C

```
⋮
DO $l_{(i-1)}$ = ....   ($s_{(i-1)}$H?I |)
   DO $outer\_l_i$ = ....   ($n$ H?I |)
      { 4pK\%?%9%/=89g$N<B9T}
      { B>$N%W%m%;%C%5$G;HMQ$5$l$k%G!<%?$NAw?.}
   ENDDO
ENDDO
⋮
```

?^4.3: 3F%W%m%;%C%5$G<B9T$5lk%W%m%0%i%`$N4pK\%?%9%/=89g$KhkI=8=

%5$,<B9T$9$k%?%9%/$,Dj5A7?G[NsMWAG$NAw?.$O!"4pK\%?%9%/=89g$KB0$9k9YF$N%?%9%/$G7W;;5l$?%G!<%?$,%Y%/%H%k2=5lk3Hr2>Dj9k(?^4.3)!#<+%W%m%;%C%5$,Dj5A$7$J$$%G!<%?$NAw?. %k!<%WFb$G;2>H7+5l$J$$%G!<%?$O!"%k!<%W$N<B9TA0K9YFAw?.9k3H$,2DG=$G$"$k!#TINPAR GO0MB82r@O$N7k2L!"Aw?.$9Y-%G!<%?$,%k!<%WFb$NIN%$%?%l!<%7%g%s$KhCFbDj5A5l$J$$$HH=L@7?>l9g!"$=$liNAw?.L?NaO9YFEv3:%k!<%W$KF~$kA0$K9T$olkh&%9%1%8%e!<%j%s%0$5$l$k!#$7$?$,CF!"@-G=8+@QjG$O!"$3li$NDL?.$O9Y$F%k!<%W$N<B9TA0$K9T$olkh&:GE,2=5l$F$$$k3H$r2>Dj$9$k!#

4.5 8+@QJ}K!$N>\:Y

K\@aGO!"$^$:@-G=8+@QjN$?$a$KJBNs2=%3%s%Q%$%i$KM?$(il$kBP>]JBNs7W;;5!8GM-$N%Q%i%a!<%?KD$$$F=RYk!#B3$$$F!"%?%9%/!"4pK\%?%9%/=89g4V$N0MB84X78$rbH$K!"DL?.%3%9%H$r0H7$?>l9g$N@-G=8+@Q$jJ}K!$K$D$$$F=RY?$"$H!"DL?.%3%9%H$N8+@QJ}K!$K$D$$$F=RYk!#:G8eK3li$r2C$(k3HKhC$F!":G=*E*$J8+@Q$j;~4V$,5aailk!#

4.5.1 BP>]JBNs7W;;5!8GM-$N%Q%i%a!<%?

@-G=8+@Qjr9kK"$?$j!"BP>]JBNs7W;;5!8GM-$N%Q%i%a!<%?!"=hM}7O0MB8$N%Q%i%a!<%?H7$F!"0J2<$NCM$,%3%s%Q%$%i$KM?$(il$F$$$kH9$k!#

- Aw?.!"<u?.L?Na$N5/F0;~4V
- Aw?.!"<u?.$KH<$&%G!<%?$N%Q%C%-%s%0!"%"%s%Q%C%-%s%0;~4V
- DL?.$N%l%$%F%s%7

1 %?%9%/$N<B9T;~4V$O!"C`<!$N<B9T;~4V$r%k!<%W$N2s?t$G3dC?;~4VH9$k!#$3 1O!"M?$($i1?%k!<%W$r%3%s%Q%$%i$,BP>]JBNs7W;;5!1 Bf8~$1$K%3%s%Q%$%k $7!"<B:]$K<B9T7?;~4VrbHK5aak!#%G!<%?J,3d$r7hDj$9$k>e$G!"J#?t$N J,3d8uJd$,$"$k>l9gGb!"$3$1iN<B9T$O1 2s9T$&$@$1Gh$$!#

4.5.2 DL?.%3%9%H$rL5;k$7$?M=Hw8+@Q

%k!<%W$NJBNs<B9T>uBV$rB*$($k<j$,$+jH7F!"0MB84X78$KCmL\$9$k!#0MB8 KO!"%U%m!<0MB8!"5U0MB8!"=PNO0MB8$,$"j=1>1KBP7F0J2<$N2 <oN`$N 0MB8$,B8:_$9$k!#

- %k!<%WFHN)0MB8

  0MB885$N<B9TJ8$H0MB8@h<B9TJ8$,%k!<%W$NF1$8%$%?%l!<%7%g%sNCfK$"$k 0MB8

- %k!<%Wl?HB0MB8($^$?$O%$%?%l!<%7%g%s4V$K8Y$,$k0MB8) [8]

  0MB885$N<B9TJ8$H0MB8@h$N<B9TJ8$,%k!<%W$N0[$J$k%$%?%l!<%7%g%sK"$k 0MB8!#

<u>0MB84X78Kh$k<B9T=g=x$N@)Ls</u> A0=R$N0MB84X78$O!"JBNs<B9TK*$$$FJ]B8$5 1kI,MW$,$"k?$a!"@-G=8+@QjN:]$KO3$N0MB84X78KhCF@)Ls$5$1$k=h M}$N<B9T=g=x$r9MN8$9kI,MW,$"$k!#FC$K!"3F%W%m%;%C%5GN<B9T>u67$rBg6I E*$KHi$($k$?$aKO!"$3$1iN0MB8KhCF@8$8$k%W%m%;%C%54V$N=hM}$N<B 9T=g=x$N@)Ls$K$D$$$F9MN8$7$J$1$1PJiJ$$!#

7+$7!"0J2<$K=RYkM}M3Kh$j!"3F%W%m%;%C%5$,B>$N%W%m%;%C%5$KBP7F<B 9T$K4X$7$FM?$($k@)Ls$H7F$O!"%k!<%Wl?HB$N%U%m!<0MB8N_$r9M$(1PNI$$!#

- %k!<%WFHN)0MB8$,M?$($k1F6A

  K\<jK!GO!"%$%?%l!<%7%g%s$r1 DN%?%9%/H7$F07$$!"%?%9%/$,=hM}$N 3djEvF$NC10L$HJk!#7?$,$C$F1 DN%$%?%l!<%7%g%sFb$N<B9TJ8O9 YFF1$8%W%m%;%C%5>e$G<B9T$5$1k?$a!"%k!<%WFHN)0MB8$O>o$KJ]$?1k!# $^$?!"$3$1iN0MB84X78$O0[$J$k%W%m%;%C%54V$KE0$k<B9T=g=x$KBP7F2? $N@)Ls$bM?$($J$$!#

- %k!<%Wl?HB0MB8$,M?$($k1F6A

  - =PNO0MB8

    =PNO0MB84X78$O!"2 DN<B9TJ8$N:8JU$NJQ?t$,F1$8G"$k>l9g$KN_ 5/3k!#JBNs2=%3%s%Q%$%i$OOwner Computes Rule Kh$k=hM}$N3d$j Ev$Fr9k?$a!"$3$1$i2 DN<B9TJ8$OI,$:F1$8%W%m%;%C%5$G<B9T$5

1k3HKJ$j!"0[$J$k%W%m%;%C%54V$G$N%G!<%?$N=q$-9~$_=g=xKD$$$F2?$i@)Ls$rM?$($J$$!#

- 5U0MB8

  5U0MB8N"$k<B9TJ8$,F1$8%W%m%;%C%5$G<B9T$5$1$k>19g$O!"%W%m%;%C%54V$G<B9T=g=x$KBP9k@)Ls$r@8$8$J$$!#01J}!"0[Jk%W%m%;%C%5$K3d$jEvFi1?<B9TJ84V$K5U0MB8$,$"$k>19g!"%3%s%Q%$%i$O;2>H$5$1$k%G!<%?$r=jM-9k%W%m%;%C%5K*$$$F!"%G!<%?$rAw?.$7$?8e$K%G!<%?$r>e=q$-9kh&$J%9%1%8%e!<%j%s%0$r9T$&$?$a!"$3$1$i$N5U0MB8$OJ]$?$1$k!#$7$?$,CF!"%G!<%?$r;2>H$9$k%W%m%;%C%5$N<B9T$N?J9T$OHo;2>H%G!<%?$r;}$D%W%m%;%C%5$N<B9T$N?J9T$K2?$i1F6A$rM?$($J$$!#

- %U%m!<0MB8

  0MB88 5$N<B9TJ8$N<B9T$,=*$o$j!"0MB88 5$N<B9TJ8$r<B9T$9$k%W%m%;%C%5$,<B9TJ8KhCF7W;;7?%G!<%?$rAw?.$7!"0MB8@h$N<B9TJ8$,$=$N%G!<%?$r<u?.$9$k!#$3$N%G!<%?$N<u?.$,9T$o1k$^$G!"0MB8@h$N<B9TJ8$N<B9T$,3+;O$G$-$J$$!#3N$?$a!"%$%?%1!<%7%g%s4V$K8Y$,$k%U%m!<0MB8$O%W%m%;%C%54V$KEOCF!"%?%9%/$N<B9T=g=x$K@)Ls$rM?$($k!#

$"$k%k!<%W$NJ}8~$KBP7F0MB88 5%$%?%1!<%7%g%s$Nd 8D@h$N%$%?%1!<%7%g%s$K0MB8@h%$%?%1!<%7%g%s$,$"$k>19g!"$=$N%k!<%WJ}8~$N0MB8$N0MB85wN%$Od G"kH8@$&!#$^$?!"%?%9%/4V$N%U%m!<0MB8$r!"%?%9%/$KBP1~9k%$%?%1!<%7%g%sFb$KB8:_9k3F%9%F!<%H%a%s%HKBP7$F5a$a$?%k!<%W1?HB%U%m!<0MB8$NOB=89gH7$FDj5A$9$k!#$^$?%?%9%/4V$N%U%m!<0MB8$N0MB85wN%$b!"%k!<%W1?HB0MB8$GMQ$$$k0MB85wN%$HF1MM$N0UL#$GMQ$$$k3HK9$k!#

<u>0MB8KhkBTA;~4V$r4^$a$?8+@Q</u> Bh$4.4.1$ @a$G=R$Y$?J}K!$G3F%?%9%/$,%W%m%;%C%5$K3djEvFi1k?$a!"3F%W%m%;%C%5$G=hM}$5$1$k%?%9%/NL$O !V+%W%m%;%C%5$,<B9T$9$k%$%s%G%C%/%9=89g!WNBg-5G5aak3H$,$G$-$k!#

01J}!"$3$1iN=hM}=g=x$,%U%m!<0MB84X7 8$KhC$F@)Ls$5$1$kH-!"$=$1i,8_$$$KI&$$$C$?4X7 8$G<B9T$,?J$ai1$F$$$kN+$r8+@Q$kI,MW$,$"$k!#

5F!"%?%9%/4V$K%U%m!<0MB8$,$"$k>19g!"$=$1iN%?%9%/$,0[$J$k%W%m%;%C%5$K3djEvFi1$F$$$k>19g$O!"DL?.$KhC$F%G!<%?$,;2>H$5$1$k!#%G!<%?$r;2>H9k%W%m%;%C%5$O!"Aw?.%W%m%;%C%5$,%G!<%?$r7W;;$7!"$=$N7k2L$rAw?.$9k^GBT?$5$1k3HKJk!#b7JBNs2=$5$1$k%k!<%W$KBP7F!"%?%9%/4V$N0MB85wN%$,d G"C?>19g!"$=$N%k!<%WKBP7Fd$ 8DA0$N%?%9%/$,<B9T$5$1$?

8e$K%G!<%?$r<u?.G-!"7W;;$r;O$ak3H,G-$k!#

7+$7!"TINPAR $N>l9g!"Bh4.4.2 @a$G=RY?h&$K!"7W;;$KhC$F@8@.$5l?%G!<%?$rB>$N%W%m%;%C%5$KAw?.$9$k>l9g!"4pK\%?%9%/=89g$KB0$9$k9YFN%?%9%/$,<B9T$5l?8e$K!"I,MW$J%G!<%?$,%Y%/%H%k2=$5lFAw?.5l$k!#$7$?$,CF!"0MB8@h$N%?%9%/$r;}$D%W%m%;%C%5$N<B9T$O!"0MB885$N%?%9%/$,B0$9$k4pK\%?%9%/=89g$N<B9T$,$9YF=*ok$^$G<B9T$r3+;O$G$-$J$$!#

$=$3$G!"%?%9%/4V$N0MB85wN%$d_i$$KBP1~9k!"4pK\%?%9%/=89g4V$N0MB85wN%$D_i$$r%9%H%j%C%W%^%$%K%s%0$Khj@8@.5l$?FbB&%k!<%W$NH?I|2s?tB(<04.1) $rMQ$$$F!"<!$N<0$KhC$FDj5A$9$k!#

$$D_i = \left\lceil \frac{d_i}{B} \right\rceil \tag{4.3}$$

$^$?!"$d_{i_{min}} = \min_i\{d_i\}$ H7$?;~!"$d_{i_{min}}$$KBP1~9k4pK\%?%9%/=89g4V$N0MB85wN%$O!"$D_{i_{min}}$$G$"$k!#3l$O!"$"$k4pK\%?%9%/=89g$+$i8+$F!"7W;;$r;O$ak?aKI,MW$J%G!<%?$,7W;;5l!"DL?.5l$k4pK\%?%9%/=89g$H$N5wN%$rI=$9!#$7$?$,CF!"$"$k4pK\%?%9%/=89g$N<B9T$O!"$=$3$+$i$D_{i_{min}}$8DA0$N4pK\%?%9%/=89g$N<B9T$,=*N;$7F+iG$J$$$H!"<B9T$r3+;O$G$-$J$$$H$$$&3HKJ$k!#

3liN4pK\%?%9%/=89g$N4X78$+$i!"JBNs2=$5lk%k!<%W$N%k!<%W1?HB0MB8$NM-L5KhCF>l9gJ,1r$7!"$=l>lN>l9gKBP9$k8+@Q$jJ}K!$r0J2<$K<($9!#$3$3$G!"JBNs2=5l$?%k!<%W$h$j30B&$N%k!<%W$NAm<B9T2s?t$N$ $r!"?^4.1$N3F%k!<%W$N<B9T2s?t$s_x$ $rMQ$$$F<!Nh$&$KDj5A9k!#

$$N = \prod_{x=1}^{i-1} s_x \tag{4.4}$$

$^$?!"$C_{TaskSet}$$O1 4pK\%?%9%/=89g$N<B9T;~4V$G$"$k!#

- <u>JBNs2=5l$k%k!<%W$K%k!<%W1?HB0MB8J7</u>

  JBNs2=5l$k%k!<%W$r0O$`%k!<%W$NM-L5$K4X$oi:!"4pK\%?%9%/=89gO9YFJBNs$K<B9T2DG=$G$"$k!#7?$,$C$F!"3F%W%m%;%C%5$O!"0MB8$K5/0x$9kBTA;~4V$,$J$/$J$k!#$hCF!"DL?.;~4V$r=|$$?<B9T;~4V$O3F%W%m%;%C%5$K3djEvFil$?4pK\%?%9%/=89g?t$,!"<0(4.2)(4.4) $GDj5A$7$?$n$!"$N$ $rMQ$$$F!"$n \times N$ $HI=$;k?$a!"0J2<$K<0$GM?$(il$k!#

$$n \times N \times C_{TaskSet} \tag{4.5}$$

- JBNs2=$5$1$k%k!<%W$K%k!<%W1?HB0MB8$"$j

  1 DN%W%m%;%C%5Fb$N4pK\%?%9%/=89g$OC`<!<B9T$5$1$k!#$n > 1$ $N>19g!"
  $l_{(i-1)}$%k!<%W$N1 H?I |$G4pK\%?%9%/=89g$,J#?t8D<B9T$5$1$k3HKJ$k!#$"
  $k4pK\%?%9%/=89g$r<B9T7?8e!"<!$N4pK\%?%9%/=89g$,<B9TG-kN$O!"
  4pK\%?%9%/=89g4V$N0MB8$,%W%m%;%C%54VKEOC$FB8:_$9k3H+$i!"0MB8
  854pK\%?%9%/=89g$N<B9T$,40N;7?8eG"$k!#01J}!"$l_1$ $+$i $l_{(i-i)}$ NG0U$N
  %k!<%WJ}8~KD$$$F$O!"0MB8$NM-L5$K4X78$J$/!"F1$8%W%m%;%C%5$G<B9T$5
  1kA0$NH?I |$N:G8e$N4pK\%?%9%/=89g$N40N;8eD>AK<B9T$,3+;O$G$-$k!#
  7?$,$C$F!"?^4.4 Nh$&$J<B9T>uBVKJ$k!#

  3N?^hj!"DL?.;~4V$r=|$$$?<B9T;~4V$N8+@QCM$O!"<!$N<0$GI=8=G-$k!#
  $?$@7P$ $O%W%m%;%C%5Bf?t$H9k!#

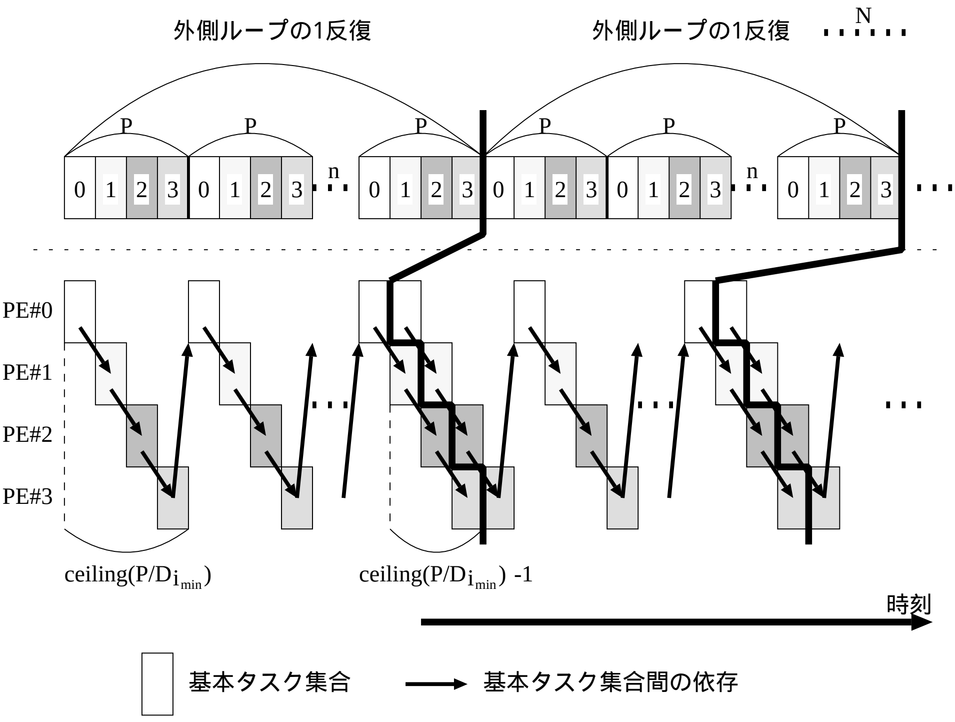
$$\left(\left\lceil\frac{P}{D_{i_{min}}}\right\rceil \times n \times N - \left(\left\lceil\frac{P}{D_{i_{min}}}\right\rceil - 1\right) \times (N-1)\right) \times C_{TaskSet} \qquad (4.6)$$

  4pK\%?%9%/=89gND_{i_{min}}$ 8D$:$D$O!"0MB84X78$KhC$F@)Ls$5$1$:$KJBNs
  <B9T$,2DG=$G$"$k!#$outer\_l_i$ N"$k1 H?I |$r=*N;9kNKI,MW$J;~4V$,!"
  $\lceil P/D_{i_{min}} \rceil \times C_{TaskSet}$ $GI=8=$5$1$k!#7?$,$C$F!"$\lceil P/D_{i_{min}} \rceil \times n \times C_{TaskSet}$
  Ol_{(i-1)}$%k!<%W$N1 H?I |$r=*N;$9kNKI,MWJ;~4V$rI=$9!#

  4pK\%?%9%/=89g4V$N0MB85wN%$N:G>.CM$D_{i_{min}}$ $,!"$P$ 0J>e$N>19g$O!"0MB8$,$"
  kKb+$+$oi:!"JBNs<B9T$,2DG=$G$"$k3H$,!"$3$N<0$KhC$FI=$5$1
  $F$$$k!#$^$?!"$((\lceil P/D_{i_{min}} \rceil - 1) \times (N-1) \times C_{TaskSet}$ $O!"$l_{(i-1)}$%k!<%W$NH?
  I |4V$G2DG=$J7W;;$N%*!<%P%i%C%W$KBP1~9k;~4VG"$k!#

4.5.3 DL?.%3%9%H$N8+@Q

Bh4.4.2 @a$G=R$Y$?$h$&$K!"Aw?.$O4pK\%?%9%/=89gFb$G7W;;$5$1$?%G!<%?$N$&
$A!"B>$N%W%m%;%C%5$G;2>H$5$1$k%G!<%?9YF,!"$=$N4pK\%?%9%/=89g$N=*N;
;~$K%Y%/%H%k2=$5$1$F$+$i9To1$k!#$7$?$,CF!"Aw?.$N5/F0$O!"3F4pK\%?%9
%/=89g4H$KH/@8$9k?$a!"$3$N%3%9%H$O!"3F%W%m%;%C%5$G<B9T$5$1$k4pK\%?
%9%/=89g$N8D?tJ,$@$12C$(kI,MW,$"$k!#$^$?!"<u?.$O!"$=$N%G!<%?$r;HMQ$9
$k4pK\%?%9%/=89g$4HKH/@8$9$k$?$a!"3N%3%9%H$b4pK\%?%9%/=89g$4HK2C
$($kI,MW$,$"$k!#$^$?!"6qBNE*$JDL?.%3%9%H$r8+@Q$k$?$aKO!"Aw?.!"<u?.$N
3F5/F02s?t$d!"3FDL?.$4HK<uEO$7$5$1$kNL$^$G5aakI,MW$,$"$k!#$3$1$i$N
3FCM$,5a$ai1$1$P!"Bh4.5.1 @a$G=R$Y$?BP>]JBNs7W;;5!8GM-$N%Q%i%a!<%?$rMx
MQ7F!"3F4pK\%?%9%/=89g4H$K2C$(J1$1$PJi$J$$DL?.%3%9%H$r5a$ak3



?^4.4: $(n > 1)$ $N;~$N<B9T>uBV

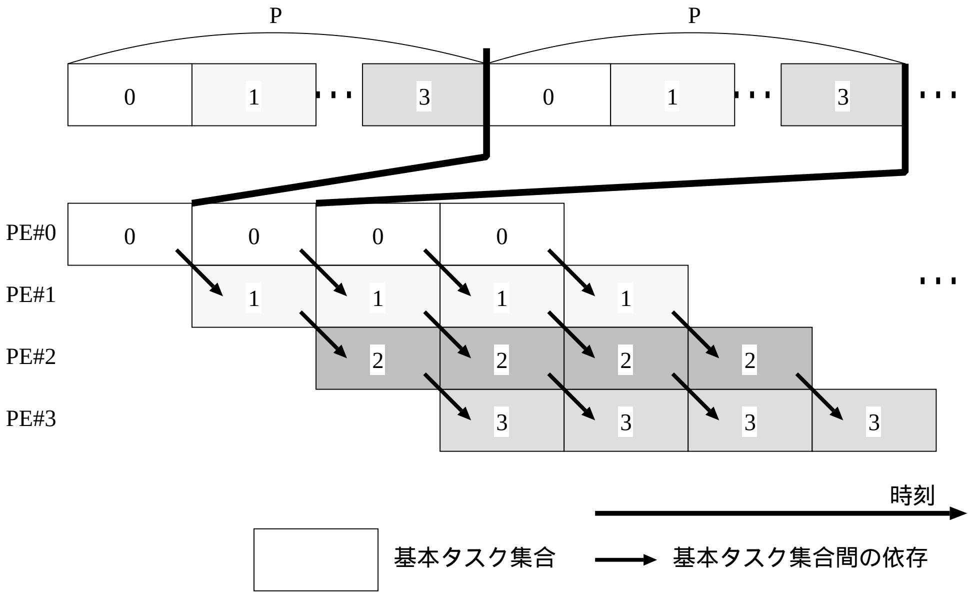
H,2DG=G"$k!#

$=$3$G!"3F4pK\%?%9%/=89g$4HNAw?.!"<u?.$N5/F02s?t$H$=$l$>$lNNLK$D$$$F$N8+@Q$j$r9T$&J}K!KD$$$F=RYk!#

Bh4.4.2@a$G=R$Y$?$h$&$K!"DL?.$O!"$=$NDL?.$KhC$F;2>H$5lkG[NsMWAG$,B>$N%?%9%/$N<B9T$GDj5A5lk+I&$+$GJ,N`G-!"$=$l$>$lKBP7$F2DG=$J%9%1%8%e!<%j%s%0$,0[$J$k!#$=3G!"$=$l$>$l$K1~$8k?aK0J2<$N8+@Q$jJ}K!rHk3HK9k!#

1. %?%9%/Fb$NBeF~J8$N1&JUN9YF$NG[NsJQ?t$KBP7F!"$=$NG[NsMWAG$,B>$N%?%9%/KhCFDj5A5lk+I&$+$r5aak!#Dj5A5l$k>l9g$K$O!"$=$NDj5A$9$k%?%9%/$H$N5wN%$d_i$$r!";2>H!"Dj5ANAH4HK5aak!#3l$O!"0MB82r@O$KhC$F5a$ak3H,2DG=G"$k!#
2. Dj5A5l$k/$5lJ$$$K1~8F0J2<$NJ}K!$G$=$NG[NsMWAG$N;2>H$KH<$&DL?.%3%9%H$r8+@Q$k!#

B>$N%?%9%/$GDj5A5l$kG[NsJQ?t $"$k%W%m%;%C%5$G<B9T$5lk%?%9%/$+$i8+$F!"%G!<%?$,;2>H5lkN$O!"JBNs2=$5lk%k!<%WJ}8~Kd_i$8D8e$N%?%9%/G"



?^4.5: $(n = 1)$ $N;~$N<B9T>uBV

$k!#$3li$,8_$$$K0[Jk%W%m%;%C%5$K3d$jEvFilF$$$k>l9g$K<B:]$NAw?.$,I,MW$HJk!#7?$,$C$F!"$"$k4pK\%?%9%/=89g$KB0$9$k9YFN%?%9%/KKBP7F!"$=$N%?%9%/$,Dj5A$7$?G[NsMWAG$r;2>H9k%?%9%/$r5a$a!"$=$li,B0$9$k%W%m%;%C%5HV9f$H!";2>H$9$k%?%9%/?t($9Jo$AAw?.NL)$r5aalPh$$!#

0lJ}!"%$%?%l!<%7%g%s4V$N0MB84X78$,uniform G"k3H+$i!"$"$k4pK\%?%9%/=89g$GDj5A5l$?%G!<%?NAwj@h%W%m%;%C%5$O!"9b!92 G"$j!"$=l>lKBP9kAw?.NL$O!"(Aw?.@h%W%m%;%C%5HV9f, DL?.NL)NAHH7F!"0J2<Nh$&$KI=8=G-$k!#$?$@$7!"<+%W%m%;%C%5HV9frCID$ H9$k!#

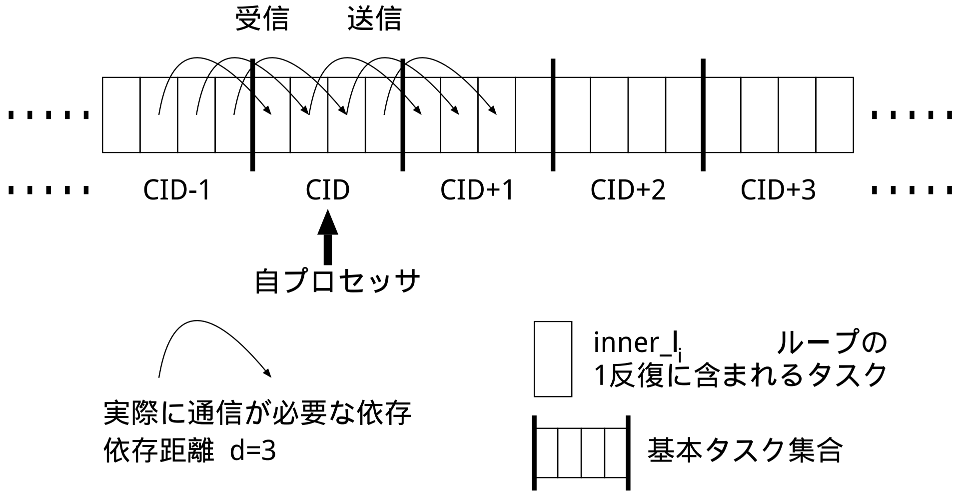
$$\left(\left(CID + \left\lceil\frac{d_i}{B}\right\rceil\right) \bmod P, V1(d_i)\right) \tag{4.7}$$

$$\left(\left(CID + \left\lceil\frac{d_i}{B}\right\rceil - 1\right) \bmod P, V2(d_i)\right) \tag{4.8}$$

$$V1(d_i) = (d_i \bmod B) \times \prod_{x=i+1}^{m} s_x \tag{4.9}$$

$$V2(d_i) = (B - (d_i \bmod B)) \times \prod_{x=i+1}^{m} s_x \tag{4.10}$$

$3$3$G!"$b7((CID + \lceil d_i/B \rceil) \bmod P) = CID$ $N>l9g!"<0$(4.7)$ KhCFI=5l$kDL?.$O<B:]KO5/F0$5$l$J$$$?$a!"8+@Q$jKO4^aJ$$!#F1MM$K!"$((CID -$



?^4.6: B>$N%?%9%/$GD j5A5l$?JQ?t$N;2>H$KH<$&DL?.

$\lceil d_i/B \rceil - 1) \bmod P) = CID$ $N>19g!"<0$(4.8)$ KhCFI=5l$kDL?.$O<B:]KO5/F0$5$l$J$$$?$a!"8+@Q$jKO4^aJ$$!#

3liNCMNAHd!"DL?.$,I,MW$KJk>r7o$O!"$CID$ NCMKhi$:!"$d_i$ KhCFN_JQ2=9k!#9JoA!"IN%W%m%;%C%5KBP7Fb!"F1$8$@1NDL?.NL$,I,MW$G$"$k3H$r<($7$F$$$k!#

?^4.6 NNcG$O!"$B = 4$!"$d_i = 3$ NNcG$"$k!#3N>19g!"$"$kCID KBP7FO!"<0(4.7) $KBP1~$9$k%W%m%;%C%5HV9f$O!"$CID + 1$!"<0(4.8) $KBP1~$9$k%W%m%;%C%5HV9f$O!"CID G"$j!"<B:]$K$O!"$CID + 1$ HV%W%m%;%C%5$K!"$V1(3)$ $@$1$N%G!<%?$,Aw?.5l$k!#

F1MM$K!"<u?.$K$D$$$F$b%G!<%?$NAw?.85%W%m%;%C%5$O9b!92$DG"$j!"$=l>lKBP9kDL?.NL$O!"<0$(4.9)(4.10)$ NV1, V2$ $rMQ$$$F!"(Aw?.85%W%m%;%C%5HV9f, DL?.NL) NAHH7F0J2<Nh$&$KI=8=G-$k!#

$$\left(\left(CID - \left\lceil \frac{d_i}{B} \right\rceil\right) \bmod P, V1(d_i)\right) \tag{4.11}$$

$$\left(\left(CID - \left\lceil \frac{d_i}{B} \right\rceil + 1\right) \bmod P, V2(d_i)\right) \tag{4.12}$$

$3$3$G!"Aw?.$N>19g$HF1MM$K!"$((CID - \lceil d_i/B \rceil) \bmod P) = CID$!"$((CID - \lceil d_i/B \rceil + 1) \bmod P) = CID$ $N>19g!"$=l>$l<0$(4.11)(4.12)$ KhCFI=5l$kDL?.$O<B:]KO5/F0$5$l$J$$$?$a!"8+@Q$jKO4^aJ$$!#

0J>e$N<0$(4.7)$ $+$i(4.12) $O!"1 DN;2>H!"Dj5ANAHKBP1~9k0MB85wN%d_i $N

0MB8KBP9kCMG$"$k!#%W%m%0%i%`Cf$KB8:_$9k9YF$N;2>H!"Dj5A$NAH$KBP1~$9kd_iKBP7F3lir9YF5aak3H$G!"l 4pK\%?%9%/=89g$4HK9Tol$kAw<u?.$N5/F02s?t$,0J2<$Nh&K7$F5a$^$k!#

- Aw?.5/F02s?t

  9YFN$d_i$$KBP7F!"<0$(4.7)(4.8)$$NAHN&$A!"Aw?.@h$,<+%W%m%;%C%5GJ$$AH$N?t

- <u?.5/F02s?t

  9YFN$d_i$$KBP7F!"<0$(4.11)(4.12)$$NAHN&$A!"Aw?.85$,<+%W%m%;%C%5GJ$$AH$N?t

>e5-$N3FAw<u?.$K$*$1$kDL?.NL$O!"<0$(4.7)$$+i(4.12)$$^GN$V1(d_i), V2(d_i)$$G$"$k!#

7?$,$C$F!"Bh$4.5.1$@a$G$"$2$?!"%3%s%Q%$%i$K4{CN$G$"$k!"DL?.$K4X$9$k%Q%i%a!<%?$Khj!"3F4pK\%?%9%/=89gFb$G9T$olkDL?.$N6qBNE*$J%3%9%H$N8+@QCM$,5aailk!#

B>$N%?%9%/$GDj5A5l$J$$G[NsJQ?t B>$N%?%9%/$GDj5A5l$J$$G[NsMWAG$N;2>H$G$"$k$?$a!"3liNG[NsJQ?t$O!"0MB8$N860xHOJC$F$$$J$$!#3N$?$a!"Bh4.2@a$G=R$Y$?0MB8$Nuniform@-$N2>Dj$+iO!"I,MWHJ$kDL?.$,!"DL?.85/@h$N%W%m%;%C%5HV9f$N:9$,Dj?t$HJkh&$J%Q%?!<%s($%7%U%H7?$NDL?.$H8FV3HK9k$)$KJk3H$O2>Dj$G$-$J$$!#7+$7!"<!$N>r7o$,@.$jN)$D>l9g$O!"DL?.%Q%?!<%s$,%7%U%H7?$NDL?.HJ$k!#

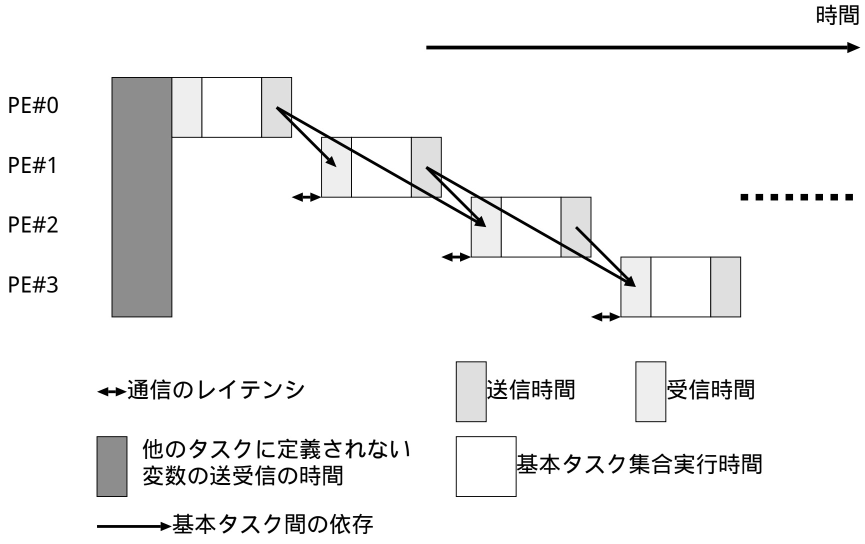
- ;2>H5l$kG[NsJQ?t$NJ,3d<!85$NE:;z<0$H!"$=$NBeF~J8$N:8JU$NG[NsJQ?t$NJ,3d<!85$NE:;z<0$,%"%U%#%s4X78$K$"$k!#
- >e5-N2DN<!85$NJ,3d$N%V%m%C%/%5%$%:$,!"$=$l$>$l$NE:;z<0$K8=lk%k!<%WJQ?t$N78?t$NHfKJCF$$$k!#

3liN>r7o$O!"%G!<%?J,3d8uJd$r5aak:]$K2DG=$J8B$jK~$?5lkh$&$K5aailk!#b7!"3liN>r7o$rK~$?5J$$>l9g!"Hs>o$KJ#;($G%3%9%H$N9b$$DL?.%Q%?!<%sKJ$k2DG=@-$,9b$$!#

7?$,$C$F!">r7o$,K~$?$5lk$+$I$&$+$G>l9gJ,$1r7!"$=$l$>$l0J2<NDLjDL?.%3%9%H$N8+@Q$r9T$&!#

1. %7%U%H7?DL?.

   $=$NDL?.Aj<j$O9b!9$2$%W%m%;%C%5$G$"$k!#3NH-$NDL?.%3%9%H$O!"4JC1N?$a!"$2$2s$NDL?.5/F0%3%9%HH7$F8+@Q$k!#$^$?!"3FDL?.NL$O!"<+%W%m



?^4.7: DL?.;~4V$r4^$a$?8+@Q;~4V

%;%C%5$,;}$D%G!<%?$NH>J,$G$"$kH9$k!#

2. $=$l0J30

   0lHIE*KO!"$"$k%W%m%;%C%5$+$i!"A4It$N%W%m%;%C%5$KBP7F$=$l$>$lAw?.!"r7J1lPJi$J$$!#7?$,$C$F!"Aw?.!"<u?.$O%W%m%;%C%5Bf?tJ,$@$15/F0$5$lk3HH7F8+@Q$k!#$^$?!"$=l>lNDL?.NL$O!";2>H$5lk%G!<%??t$r%W%m%;%C%5Bf?t$G3dC?NL$:$DG"kH7F8+@Q$k!#

3l$i!"Aw?.!"<u?.$N%3%9%H$O!"8+@Q$jBP>]$N%k!<%W$KF~$kA0$KIN%W%m%;%C%5Kb0lMMKI,MWG$"$j!"$3$3$G5a$ail$?%3%9%H$O!"8+@QKh$kA4<B9T;~4V$K$=$N$^$^2C;;5l$k!#

4.5.4 :G=*E*$J8+@Q;~4V$N;;=P

5F!":G=*E*$J8+@Q$j;~4V$O!"DL?.;~4V$rL5;k7F0MB84X78$+$i8+@QC?;~4V$KDL?.;~4V$r2C$($F5aailk!#3liN2C$(J}$K$D$$$F=RYk!#

- DL?.$N%3%9%H

  B>$N%?%9%/$GDj5A5l$kG[NsJQ?t$NAw<u?.$N;~4V$O!"3F4pK\%?%9%/=89g4H$K2C$(il$k!#$7$?$,CF!"$C_{TaskSet}$$NCM$r!"$3li$NAw<u?.$N;~4V$r2C$($?CM$H7F7W;;9lPh$$!#

  B>$N%?%9%/$GDj5A5l$J$$G[NsJQ?t$NAw<u?.$N;~4V$O!"%k!<%W$N<B9TA0$K9T$olkNG!"3N;~4V$NJ,$@$1A4BN$N8+@Q$j;~4V$K2C$($k!#

- DL?.$N%l%$%F%s%7

3FDL? .KBP 7$FH/@8$ 9$k%1%$%F%s%7$O! "4pK\%?%9%/=89g4V$N0MB85wN%$,:G
$b>.$5$ $0MB8$KhG$FH/@8$ 9$kDL?.$KD $$FN_9MN8$ 9$1$Ph $! #$ 7$?$,$C
$F! "3F4pK\%?%9%/=89g$ 4HKH/@8$ 9$kDL? .$N$&$A! ":G$b0MB85wN%$,>.5 $
bN$K5/0x$ 7$FH/@8$ 9$kDL? .$NDL? .NL$KBP1~$ 9$k%1%$%F%s%7$r! "$C_{TaskSet}$
NCMK2C$ ($k! #

0J>er^Ha$??^$r?^4.7 $K< ($9! #

4.6 G[Ns<!85J,3dE,@-EY$rMQ$$$?J,3d<!85$N7hDj

A0@a$^$G$G=R$Y$?@-G=8+@Q$j$,IT2DG=$G$"$k>19g$NJ,3d<!85$N7hDjJ}K!KD
$$$F=RYk! #K\7hDjK!$O! "%k!<%W1?HB%U%m!<0MB8>pJs$H! "3F%k!<%W$N<B9T2s?t
$+$iJ,3d<!85$N7hDj$r9T$&! #

@-G=8+@QK!$N@bL@$G=RY?h&$K! "$"$k%k!<%W$K4X7F%$%?%1!<%7%g%s4V$K
8Y$,$k%U%m!<0MB8$,$"$k>19g! "$=$N%U%m!<0MB8$KhC$F! "$"$k%W%m%;%C%5$N<B
9T$,! "B>$N%W%m%;%C%5$N<B9T$r%V%m%C%/9k$?$a! "$=$N%k!<%W$OJBNs2=$7$J$$
J}$,NI$$! #5U$K! "%$%?%1!<%7%g%s4V$K8Y$,$k%U%m!<0MB8$,$J$$>19g$O! "%U%m!<
0MB8$,$"$k%k!<%W$rJBNs2=9k>19gKHfY$F! "$=$N%k!<%W$rJBNs2=9k$HJBNs
@-$rCj=P$G$-$k$?$a! "$=$N%k!<%W$rJBNs2=$9Y-G"$k! #

0lJ}! "Owner Computes Rule KhCFJBNs2=9k>19g! "BeF~J8$N:8JU$K8=$1
$kG[NsJQ?t$N! "J,3d$5$1$k<!85$NE:;z<0$K8=$1$kJQ?t$r%k!<%WJQ?tH9$k%k!<
%WN&$A! "$=$NBeF~J8$r0O$`%k!<%W$,JBNs2=$5$1$k! #

$3$1iNJ,3d$5$1$kG[Ns<!85$HJBNs2=$5$1$k%k!<%W$N4X78$rMxMQ$7! "JBNs2=
9Y$-($9Y-GJ$$) %k!<%W$rJBNs2=$9$k($7$J$$) h&$JG[NsJQ?t$NJ,3d<!85
$r5a$ak3H,2DG=G"$k! #

0J2<$K6qBNE*$JJ}K!$r<($9! #J*! "$"$k%k!<%WKBP7$F! "%$%?%1!<%7%g%s4V
$K8Y$,$k%U%m!<0MB8$NM-L5KhCF5aai1k! "$=$N%k!<%W$NJBNs<B9T$N2DG=
@-$r! "$=$N%k!<%W$KBP9k%k!<%WJ,3dE,@-EY! "%k!<%WJ,3dE,@-EY$+$i5aai$1
$k3FG[Ns<!85$rJ,3d9Y$-$G$"$k$+$I$&$+$rI=$9CM$rG[Ns<!85J,3dE,@-EY$H8F
V3HK9k! #

1. %k!<%WJ,3dE,@-EY$N7W;;
   $^$:! "3F%k!<%WKBP7$F%$%?%1!<%7%g%s4V$K8Y$,$k0MB82r@O[8] $r9T$J$&! #
   <!$K! "%U%m!<0MB8$,$"$k%k!<%WKBP7FO! "$=$N%k!<%W$N<B9T$,4 0N;9k
   ;~4VH7$F:G0-$N%1!<%9$rA[Dj$7! "$=$N%k!<%W$rC`<!<B9T$7$?;~$N<B9T;~

4V$rH?1G$7$?CM$r%Z%J%k%F%#H7FD1k!#$3$3GO$=$NCMH7$F!"$=$N%k!<%W$K0O$^$lk9YF$N%k!<%W%M%9%H$KhC$F%k!<%W%\%G%#$,<B9T5l$k2s?t$r%Z%J%k%F%#H7$F:NMQ$9$k!#$9Jo$A!"%k!<%WJ,3dE,@-EY$H7F(%k!<%W%\%G%#$N<B9T$5lk2s?t) ×(-1) rD1k!#$=$10J30$N%k!<%W$KBP7F$O0 $rJ,3dE,@-EY$H7FD1$k!#

2. G[Ns<!85J,3dE,@-EY$N7W;;

   %k!<%W%\%G%#$N:8JU$K8=lkG[NsJQ?t$N3F<!85$KBP7F!"$=$NE:;z<0$K8=$l$k%k!<%WJQ?t$N%k!<%W$N%k!<%WJ,3dE,@-EY$r!"$=$N<!85$NG[Ns<!85J,3dE,@-EY$H7F2C$($k!#

3. G[Ns<!85%"%i%$%s%a%s%H$5l?%0%k!<%WKBP9$kJ,3dE,@-EY$N7W;;

   G[Ns<!85%"%i%$%s%a%s%H$KhC$F%0%k!<%WJ,$1$5$l$?3F%0%k!<%W$KBP7F!"$=$N%0%k!<%W$KB0$9$kG[Ns<!85$NG[Ns<!85J,3dE,@-EY$NAmOB$r5a$a!"$=NCMr$=$N%0%k!<%W$NJ,3dE,@-EY$H9k!#

>e5-$N%0%k!<%W$KBP7FD1il$?J,3dE,@-EY$NCf$G:GBgCM$rHk%0%k!<%W$rJ,3d<!85$H7F:NMQ9k!#

G[Ns<!85J,3dE,@-EYKhCF5aakJ,3d<!85$O!"E,MQ2DG=$J%W%m%0%i%`$NHO0O$,9-$$$H$$$&FCD'$r;}$D0lJ}!"@53N$JDL?.NL$d!"1i;;NLJI$N8+@Q$j$r9T$($J$$$?$a!"I,$:7b:GE,$J%G!<%?J,3d$r5aakHO8BiJ$$!#7+$7!"2r@OBP>]$N%k!<%WN&$A!"%$%?%l!<%7%g%s4V$K8Y$,$k0MB84X78$,$J$$$h$&$J%k!<%W$,B8:_$9$k>l9g!"$=$N%k!<%W$rM%@hE*$KJ,3d$9kh$&7hDj$9$k!#$3$N7hDj$OBEEvJbNH9M$($ilk!#

5 I>2A

K\>OGO!"Ds0F9k%G!<%?J,3d7hDj<jK!$NM-8z@-$r<($9!#

$^$:!"G[Ns<!85%"%i%$%a%s%H!"%V%m%C%/%5%$%:Hf$N7hDj<jK!$,!"K\<jK!GE,EvG$"$k$+$I$&$+$rI>2A$KhC$F3NG'$9$k!#

5i$K!"Bh4 >O$G=RY?8+@QJ}K!$,E,MQ2DG=$J%W%m%0%i%`KBP7$F!"G[Ns<!85J,3dE,@-EY$G5aailkJ,3dhj$b!"8+@Q$KhC$F5a$ail$kJ,3d<!85$,:GE,JbNKJk$+$I$&$+$rI>2A$9$k!#

5.1 G[Ns<!85%"%i%$%a%s%H7hDj$NM-8z@-

G[Ns<!85%"%i%$%a%s%H7hDj<jK!$NI>2A$r9T$&$?$a$K!"livermore kernel No.8 $N%W%m%0%i%'(?^5.1) $rMQ$$$k!#3N%W%m%0%i%'KBP7!"Ds0F<jK!KhCF5a$^$k%G!<%?J,3d$H!"$=$10J30$N%G!<%?J,3d$r;XDj$7!"JBNs2=%3%s%Q%$%iTINPAR KhCFJBNs2=7?%W%m%0%i%'$r!"AP1000 $G<B9T$7$?7k2L$rHf3S9k!#J*!"AP1000 $O64 Bf9=@.$G!"I>2AGO!"1!"2!"4!"8!"16!"32!"64 BfN=l>lNBf?t$G<B9T$7$?;~4V$r7WB,7?!#

Ds0F<jK!KhCF5a$^$k%G!<%?J,3d M?$($il?%W%m%0%i%'KBP7$F!"$^$:!"G[Ns<!85%"%i%$%a%s%H$N7hDj$N%9%F%C%WGO!"?^5.2 $G<($5lkCAG $r:n@.$9$k!#$3$NCAG $+$i%"%i%$%s%a%s%H$H7F!"DU1_1!"DU2_1!"DU3_1!"U1_2!"U2_2!"U3_2N3F G[Ns<!85$rCj>]%W%m%;%C%5$NF1$8<!85$K@0Ns9k$H$$$&7k2L$,5a$ail$k!#$^$?!"%V%m%C%/%5%$%:Hf7hDj$N%9%F%C%W$G$O!">e5-$NG[Ns<!85Hb9Y$F1:1 $H5a$ail$k!#$5iK!"%V%m%C%/%5%$%:7hDj$N%9%F%C%WGO!"3FG[Ns<!85$N%0%k!<%W$H$b%V%m%C%/J,3d$HJkh&$J%V%m%C%/%5%$%:$,5a$ail$k!#

5F!"3N%W%m%0%i%'KBP9$kJ,3d<!85$H7FA*Pl$kG[Ns<!85$N%0%k!<%W$O!"G[Ns<!85%"%i%$%a%s%H$G5a$^C?DU1_1!"DU2_1!"DU3_1!"U1_2!"U2_2!"U$3_2$$@1G$"$k!#=>CF!"K\%W%m%0%i%'KBP7FO!"J,3d<!857hDj$N%9%F%C%W$G$O!"8+@Q$j$dG[Ns<!85J,3dE,@-EY$JIr5aak3HJ/!">e5-$NG[Ns<!85$N%0%k!<%W$rJ,3d$9kH$$$&7k2L$,F@$ilk!#

7k6I!"K\<jK!KhCF!"DU1_1!"DU2_1!"DU3_1!"U1_2!"U2_2!"U3_2N3F<!85$r%V%m%C%/J,3d$9kH$$$&h&$K7hDj$5lk!#3NJ,3d$rJ,3d1 $H8F$V3HK9$k!#

Ds0F<jK!$G5a$^$k%G!<%?J,3d0J30$NJ,3d 3lKBP7!"K\<jK!$G5a$^$kG[Ns<!85%"%i%$%a%s%H$H0[$J$kG[Ns<!85%"%i%$%a%s%H$+$i5a$^$kJ,3dH7F, DU1_1$!"DU$2_1$!"DU$3_1$!"U$1_1$!"U$2_1$!"U$3_1$ $rCj>]%W%m%;%C%5$NF1$8<!85$K3djEvF!"$=$N<!85$r%V%m%C%/J,3d$7$?$bNr9M$($k!#3NJ,3d$rJ,3d2 $H8F$V3HK9$k!#

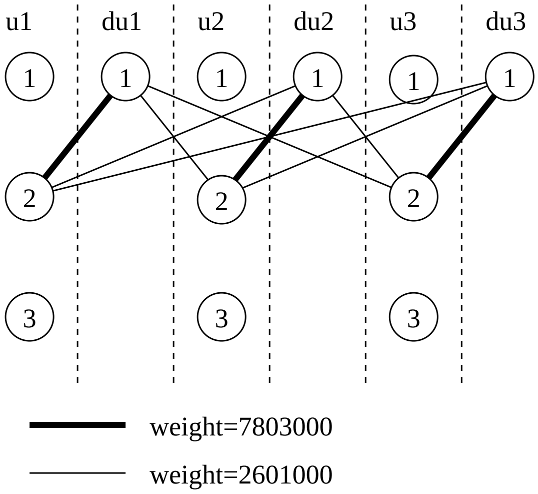
5.1.1 <BB,7k2L

3FJ,3dKBP9$k<B9T;~4V$HBf?t8z2L$rI=5.1 $K<($9!#

3N7k2L!"K\<jK!KhCFF@il$?J,3d1 $,L@$+$K@-G=$,NI$$$3H,J,$+$k!#J,3d1 GO!"64 Bf$N>l9gBf?t8z2L$,41.0 H^$:$^$:$N@-G=$r<($7$F$$$kNKBP$7!"J,3d2 GO!"Bf?t$,A}$(k?SK<B9T;~4V$,A}$($F$$/$H$$$&7k2LHJC?!#J,3d2 $N>l9g!"G[Ns<!85%"%i%$%a%s%H$,E,@Z$KDjailF$$$J$$$?$a!"$[HsIN%$%?%l!<%7%g%s$N<B9T$K!"1&JU$N9YFN%G!<%?$NAw<u?.$,I,MWHJ

```
PROGRAM KERN08
PARAMETER ( NX=512 , NY=512 )
DO kx = 2,NX-1
  DO ky = 2,NY-1
    DU1(ky)=U1(kx,ky+1,nl1)  -  U1(kx,ky-1,nl1)
    DU2(ky)=U2(kx,ky+1,nl1)  -  U2(kx,ky-1,nl1)
    DU3(ky)=U3(kx,ky+1,nl1)  -  U3(kx,ky-1,nl1)
    U1(kx,ky,nl2)=U1(kx,ky,nl1) +A11*DU1(ky) +A12*DU2(ky) +A13*DU3(ky)
o           + SIG*(U1(kx+1,ky,nl1) -2.*U1(kx,ky,nl1) +U1(kx-1,ky,nl1))
    U2(kx,ky,nl2)=U2(kx,ky,nl1) +A21*DU1(ky) +A22*DU2(ky) +A23*DU3(ky)
o           + SIG*(U2(kx+1,ky,nl1) -2.*U2(kx,ky,nl1) +U2(kx-1,ky,nl1))
    U3(kx,ky,nl2)=U3(kx,ky,nl1) +A31*DU1(ky) +A32*DU2(ky) +A33*DU3(ky)
o           + SIG*(U3(kx+1,ky,nl1) -2.*U3(kx,ky,nl1) +U3(kx-1,ky,nl1))
  ENDDO
ENDDO
end
```

?^5.1: livermore kernel No.8 %=!<%9%j%9%H



?^5.2: livermore kernel No.8 KBP9$kCAG

$j!"$3l,860xHJCF@-G=$,Dc2<$7$F$$$kbN$H9M$(il$k!#

5.2 @-G=8+@QKh$kJ,3d<!857hDj$NM-8z@-

J,3d<!85$N7hDj$N%9%F%C%WK*$$$F!"@-G=8+@QKhCF5a$^$kJ,3d<!85$,!"G[Ns<!85J,3dE,@-EY$Khk7hDj<jK!$G5a$^$kJ,3d$h$j:GE,$JJ,3d$r5a$ak3H,2DG=G"k+I&$+$rlivermore kernel No.23 KhCFI>2A9k!#?^5.3 $K%W%m%0%i%`%j%9%H$r<($9!#

5.2.1 Ds0F<jK!KhCF5a$^$k%G!<%?J,3d

%W%m%0%i%`%j%9%H$+$i!"G[Ns<!85%"%i%$%a%s%H$O3FJQ?t$N1<!85L\$I$&$7!"2<!85L\$I$&$7r=l>$l%0%k!<%W$H9k$H$$$&7k2L$,5a$ail$k!#%V%m%C%/%5

	J,3d1		J,3d2	
Bf?t	<B9T;~4V(sec)	Bf?t8z2L	<B9T;~4V(sec)	Bf?t8z2L
1	22.27		22.27	
2	11.82	1.8	129.25	0.172
4	6.03	3.6	245.23	0.090
8	3.11	7.1	460.20	0.048
16	1.64	13.5	966.31	0.230
32	0.91	24.3	1950.24	0.114
64	0.54	41.0	3951.16	0.005

I=5.1: livermore kernel No.8 <B9T7k2L

```
      PROGRAM KERN23
      PARAMETER ( JN=512 , KN=512 ,NTIME=...)
      DO t= 0, NTIME-1
       DO j= 2,JN-1
        DO k= 2,KN-1
          QA= ZA(k,j+1)*ZR(k,j) +ZA(k,j-1)*ZB(k,j) +
     o        ZA(k+1,j)*ZU(k,j) +ZA(k-1,j)*ZV(k,j) +ZZ(k,j)
          ZA(k,j)= ZA(k,j) +.175*(QA -ZA(k,j))
        ENDDO
       ENDDO
      ENDDO
      end
```

?^5.3: livermore kernel No.23 %=!<%9%j%9%H

%$%:Hf$O!"3F%0%k!<%WHb!"9Y$F1:1$H5a$^$k!#$^$?!"%V%m%C%/%5%$%:$O!"2DG=$J8B$jBg$-$J%V%m%C%/%5%$%:$rHk$H$$$&7hDj$,$J5l!"$$$:lN%0%k!<%W$rJ,3d$9$k:]$b%V%m%C%/J,3dHJ$k!#

5F!"J,3d<!85$N7hDj$K$"$?$jK\<jK!$G@8@.5l$kJ,3d<!85$N8uJd$O!"0J2<$N2DG$"$k!#

1. 3F<!85$N1<!85L\$r%V%m%C%/J,3d(J,3d3$H8F$V)
2. 3F<!85$N2<!85L\$r%V%m%C%/J,3d(J,3d4$H8F$V)

>e5-$N3FJ,3d$KBP7F$O!"%W%m%0%i%`Cf$NDj?tNTIME$,M?$(il$F$$$l$P!"K\<jK!$N@-G=8+@QIt$,E,MQ$G$-$k!#

$=$3$G!"$5$^$6$^$JNTIME$NCM!"%W%m%;%C%5Bf?t$KBP7FDs0F7?@-G=8+@QrE,MQ7!"$=$N8+@QCM$r7W;;$7$?!#$=$N%0%i%U$rIUO?$N?^A.2!"A.4!"A.6!"A.8!"A.10!"A.12$K<($9!#

8+@QGO!"1 BfGN<B9T;~4V$r%k!<%W$N<B9T2s?t$G3d$C$?CM$r%?%9%/$N%3%9%H$H7F;HMQ7?!#

3liN7k2L$+$i!"@-G=8+@QGO!":G30B&$Nt %k!<%W$NH?I|2s?tNTIME NCMKhC$F:GE,$JJ,3d$,0[$Jk3HrM=B,7F$$$k!#9JoA!"NTIME NCM,>.$5$$>l9g$O!"J,3d3 $r!"NTIME NCM,Bg$-$$>l9g$O!"J,3d4 $r:NMQ$9k3HKJk!#3liNBg>.4X78$,F~$l49okNTIME NCMr!"I=5.3 $K<($9!#

5.2.2 G[Ns<!85J,3dE,@-EYKh$kJ,3d<!85$N7hDj

G[Ns<!85J,3dE,@-EYKh$kJ,3d<!85$N7hDj<jK!$O!"%W%m%0%i%`$K$h$i$:E,MQ2DG=$G$"$k!#$=$3$GHf3S$N$?$a!"livermore kernel No.23 $N%W%m%0%i%`$KBP7FG[Ns<!85J,3dE,@-EY$r5a$a!"$=$3$+$i5a$^$kJ,3d<!85KD$$$F0J2<$K=R$Y$k!#

3N%W%m%0%i%`$O!"%k!<%W$NIN%M%9%HJ}8~KBP7Fb%k!<%W1?HB%U%m!<0MB8$,$"k3H,0MB82r@OKhCFJ,$+$k!#3N$?$a!"%k!<%WJ,3dE,@-EYH7$F!"t %k!<%W$KBP7F$O!"$-$NTIME$\times 510 \times 510$!"j %k!<%W$KBP7FO-510 \times 510$!"k %k!<%W$KBP7F$O!"$-510$ $H5a$^$k!#$^$?!"G[Ns<!85J,3dE,@-EY$O!"ZA$_1$ $,$-510$!"ZA$_2$ $,$-510 \times 510$ HJ$k!#=>$C$F!"J,3d<!85$H7F$O!"NTIME NCMK$+$+oi$:!"3F<!85$N1 <!85L\$rJ,3d9k(J,3d3) $H$$$&7k2L$,F@$ilk!#

5.2.3 @-G=8+@Q$H<BB,7k2L$NHf3S

%W%m%;%C%5Bf?t4H$K!"NTIME NCMHAP1000 GN<B9T;~4V$N4X78$r%0%i%U2=7?bN$rIUO?$N?^A.1!"A.3!"A.5!"A.7!"A.9!"A.11 $K<($9!#

@-G=8+@Q$O!"J,3d<!85$N0c$$$H!"NTIME $NJQ2=$KH<$&<B9T;~4V$NJQ2=$N798~$,@5$7$/B*$($ilF$$$k3H$,J,$+$k!#J,3d3 $HJ,3d4 $N79$-$NBg>.4X78$ODM$K<BB,$NbN$HH0lCW$7$F$$$k!#

8+@QK*1k<B9T;~4V$NBg>.4X78$,F~$l49$o$kE@(I=5.3) $O!"<BB,$K$*$1kb$N(I=5.2) $HHf3S$7$F!"@53N$K5a$^$C$F$$$kHO8@$$$Fq$$$,!"t %k!<%W$NH?I|2s?t$r==J,Bg$-$/7?>l9g$d!"==J,$K>.5/7?>l9gKO!"2 DNJ,3d$N<B9T;~4V$NBg>.4X78$N798~$r@5$7$/B*$($ilF$$$k$?$a!":GE,$JJ,3d<!85$r5aak3H$,2DG=$HJk!#

0lJ}!"G[Ns<!85J,3dE,@-EYKhCF5aailkJ,3d<!85$O!"NTIME NCMKhi$:!"J,3d3 $,A*$Plk!#3N$?$a!"NTIME NCM,Bg$-$/JC$?>l9g$O!"ITE,@Z$J%G!<%?J,3d$,A*Plk3HKJk!#

%W%m%;%C%5Bf?t	2	4	8	16	32	64
NTIME	7	17	26	27	29	30

I=5.2: <BB,K*1k<B9T;~4V$NBg>.4X78$,F~$l49$o$kE@

%W%m%;%C%5Bf?t	2	4	8	16	32	64
NTIME	12	18	22	23	24	25

I=5.3: 8+@QK*1k<B9T;~4V$NBg>.4X78$,F~$l49$o$kE@

5.2.4 9M;!

B,Dj7k2L!"@-G=8+@Q$N$$$:lNJ}K!Gb!"NTIME $N0c$$$KhC$F:GE,$J%G!<%?J,3d$,0[$Jk3H,H=L@7?!#J<3Nh&$J:9$,=PkN$+$r9M;!9k!#

J,3d3 $O!"3F<!85$N1<!85L\$rJ,3d9k$?$a!"k %k!<%W$,JBNs2=$5lk!#3N$?$a!"4pK\%?%9%/=89g$O!"(k %k!<%W$NH?I|2s?t)/(%W%m%;%C%5Bf?t) $N%?%9%/$HJk!#0lJ}J,3d4 $O!"2<!85L\$rJ,3d9k$?$a!"j %k!<%W$,JBNs2=$5lk!#3N$?$a!"4pK\%?%9%/=89g$O!"(j %k!<%W$NH?I|2s?t)/(%W%m%;%C%5Bf?t) × (k %k!<%W$NH?I|2s?t) HJ$k!#$3Nh$&$K!"JBNs2=5l$k%k!<%W$Khj!"4pK\%?%9%/=89g$NN3EY$,0[Jk!#

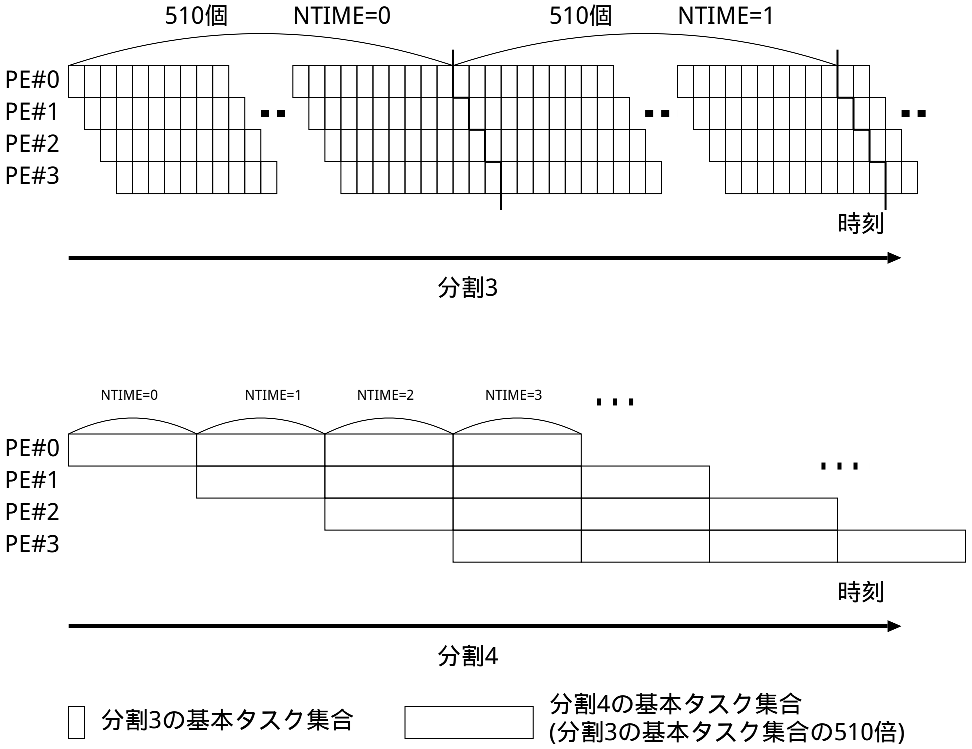
$^$?!"3F4pK\%?%9%/=89g4V$KB8:_$9$k%U%m!<0MB8$O!"IAiNJ,3d$N>l9g$b1G"$k!#$3liN3H+$i!"JBNs<B9T;~$N<B9T>uBV$O!"?^5.4 Nh$&$KJk!#$?$@$7!"3N?^KO!"DL?.%3%9%H$O4^$^lF$$$J$$!#

3N?^$+$i!"NTIME $NBg>.$KhC$F!"0J2<$N3H$,9M$(il$k!#

- NTIME $,%W%m%;%C%5Bf?t$HHf3S7F==J,>.$5$$;~

  J,3d4 $O!"4pK\%?%9%/=89g$NN3EY$,Bg$-$$$?$a!"DL?.5/F02s?t$,Dc8:5lFO$$$k$,!"JBNs@-$,Cj=PG-$F$$$J$$!#J,3d3 $O!"4pK\%?%9%/=89g$NN3EY$,>.$5$$$?$a!"DL?.5/F02s?t$,B?$/$Jk,!"Aa$$;~9o$+$iA4%W%m%;%C%5$,2TF/$9k?$a!"JBNs@-$rCj=PG-$F$$$k!#
- NTIME $,%W%m%;%C%5Bf?t$HHf3S7F==J,Bg$-$$;~

  J,3d4 $O!"A4%W%m%;%C%5$,2TF/9k$^$G$K;~4V$,$+$+kbNN!"NTIME $,==J,$KBg$-$$$?$a!"$3li$N;~4V$OL5;kG-kh$&$KJk!#$^$?!"4pK\%?%9%/=89g$NN3EY$,Bg$-$$$?$a!"DL?.$N5/F02s?t$OJ,3d3 HHfYFDc/M^$($ilk!#0lJ}!"J,3d3 $N>l9g!"A4%W%m%;%C%5$,2TF/9k$^$GK+$+$k;~4V$,C;$$$,!"NTIME $,==J,$KBg$-$$$?$a!"$3lKhkJ,3d4 HN:9$O!"A4BN



?^5.4: 3FJ,3dGN<B9T>uBV(%W%m%;%C%5?t4 $N>l9g)

$N<B9T;~4V$+$i8+$FL5;kG-kh$&$KJk!#7+$7!"4pK\%?%9%/=89g$NN3EY$N>.$5$5$N$?$a!"NTIME $NA}2C$KH<$&DL?.5/F02s?t$NA}2C$,!"J,3d4 $HHf3S$7FBg-$/$J$k!#

@-G=8+@Qr9k3HKhC$F!"$3li$N798~$rB*$($k3H$,2DG=$HJC$?!#G[Ns<!85J,3dE,@-EY$G$O!"DL?.$H1i;;$N%3%9%H$NHf$r9MN8$KF~l:!"0MB84X78$@$1$+$i5aak$?$a!"I,$:$7$b:GE,$J%G!<%?J,3d<!85$,5a$^kH$O8B$i$J$$!#

6 $*$ojK

K\9FGO!"JBNs2=%3%s%Q%$%i$K$*$1$k<+F0%G!<%?J,3d7hDj<jK!$NDs0F$r9T$C$?!#%G!<%?J,3d7hDj$O!"(1) G[Ns<!85%"%i%$%s%a%s%H$N7hDj!"(2) %V%m%C%/%5%$%:Hf$N7hDj$H$$$&G[NsJQ?t4V$NJ,3d$K4X9kAjBPE*$J4X78$r!"Cj>]%W%m%;%C%5$K3d$jEvFk3H$G5a$a!"5iK3li$NCj>]%W%m%;%C%5$KBP$7!"(3) %V%m%C%/%5%$%:$N7hDj!"(4) J,3d<!85$N7hDj $r9T$$!":G=*E*$J%G!<%?J,3d$r5aak!#

G[Ns<!85%"%i%$%s%a%s%H$N7hDj!"%V%m%C%/%5%$%:Hf$N7hDj$O!"%=!<%9%W%m%0%i%`$r2r@O$7!"$=l>$lCAG!"BCG $H8F$Plk%0%i%U$r:n@.$7$F%0%i%U$NLd

BjH7$F2r$/J}K!$r<($7$?!#$3$NJ}K!$O!"**PARADIGM** K*1k7hDjJ}K!$H $[$\F1$8$G$"$k$,!"3F%0%i%U$N%(%C%8$N=E$_E1$r!"$=$N%(%C%8$rF3$$$?BeF~ J8$N<B9T2s?t$NAmOBH7$F5a$ak3HG!"=E$_7W;;$N%3%9%H$r7Z8:$7$?!#

$^$?!"J,3d<!85$N7hDj$G$O!"@-G=8+@Q$j$rMxMQ$7$?J}K!$H!"G[Ns<!85J,3dE, @-EY$N7W;;$KhkJ,3d<!85$N7hDjJ}K!$N2DNJ}K!$r<($7$?!#

@-G=8+@QjrMxMQ7?7hDj$r9T$&<jK!GO!"E,MQ2DG=$J%W%m%0%i%`$NHO0O$, 8B$ilk$,!"DL?.%3%9%H!"1i;;%3%9%H$r9MN8$KF~lFJ,3d<!85$r7hDj$9k?$a!" hj:GE,$J%G!<%?J,3d$r5aak3H$,2DG=G"$k!#

0lJ}!"G[Ns<!85J,3dE,@-EYKh$k<jK!$O!"IN%k!<%W$rJBNs2=$9Y-G"$k $+$H$$$&>pJs$r0MB82r@O7k2L$+$i5a$a!"$=$N%k!<%W$rJBNs2=$G$-$kh&$JG[Ns $NJ,3d<!85$r5aak!#3N<jK!$O!"E,MQ$G$-$kHO0O$,9-$$$,!"DL?.%3%9%H$H1i ;;%3%9%H$NHf$r9MN8$KF~lF$$$J$$$?$a!"I,$:7b:GE,$J%G!<%?J,3d$,5a$^$k HO8BiJ$$!#

3l$O!"livermore kernel No.23 $N%W%m%0%i%`$KBP7F$O!"@-G=8+@Q$j$rMxMQ 7?<jK!KhCF5a$^$kJ,3d$,!"G[Ns<!85J,3dE,@-EYKhCF5a$^$kJ,3dhj $b:GE,$HJC$?!#$3$N2$D$N<jK!$rAH$_9g$o$;$k3HKh$j!"<+F0%G!<%?J,3d 7hDj$r9T$($kHO0O$r69$ak3HJ$/!"2DG=$J>l9gKO!"hj:GE,$J%G!<%?J,3d 7hDj$,2DG=HJC?!#

:#8e$N2]Bj$H7F!"J,3d<!85$,2<!850J>e2DG=$J>l9gXNBP1~$d!"@-G=8+@Q E,MQ2DG=HO0O$N3HBg!"5i$K!"@-G=8+@Q$j$N@:EY$N8~>eJI$,5s$2il$k!#

<UK-

:G8e$K!"K\8&5f$N5!2q$rM?$($F2<$5$j!"E,@Z$J8f;XF3$r;r$oC?IYEDbC<#65 <x$K?<?S$J<U0U$rI=$7$^$9!#

$^$?!"5.=EJ4=u8@$rD:$$$??9bC0lO:=u65<x!"8^Eg@5M5=u<j$K?<$/46<UCW$7 $^$9!#

5i$K!"K\8&5f$KBP7FF|:"$+$i8fF$O@D:$$$?7&ED>;;K;a!"EDCfMxI';a!"ED Cf?5;J;a!"JBSK5~ETBg3X9)3XIt>pJs9)3X2JIYED8&5f<<$N=t;a$K46<UCW7^$9!#

;29MJ88%

[1] ;05H0jIW, A0;39@Fs, 8eF#?5Li, ?9bC0lO:, CfEg9@, IYEDbC<#: %a%C%;!<%88r 497?JBNs7W;;5!N?aNJBNs2=%3%s%Q%$%i TINPAR, JBNs=hM}%7%s%]%8%&

%`JSPP'95, pp. 51–58 (1995).

[2] High Performance Fortran Forum: High Performance Fortran Language Specification (Ver. 2.0), Technical report, Rice University (1997).

[3] Bixby, R., Kennedy, K. and Kremer, U.: Automatic Data Layout Using 0-1 Integer Programming, *Proceedings of the International Conference on Parallel Architectures and Compilation Technique(PACT94)*, Montreal, Canada, pp. 111–122 (1994). also available as technical report CRPC-TR93349-S by Rice Univ.

[4] Kennedy, K. and Kremer, U.: Automatic Data Layout for High Performance Fortran (1995).

[5] ?JF#C#Li, 4d2<1Q=S, EZHn<B5W, Gk86=c0l, 6b>k%7%g!<%s: Twisted Data Layout, JBNs=hM}%7%s%]%8%&%`JSPP'94, pp. 161–168 (1994).

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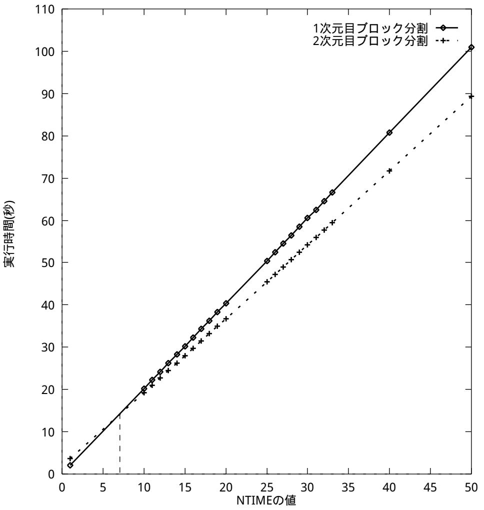
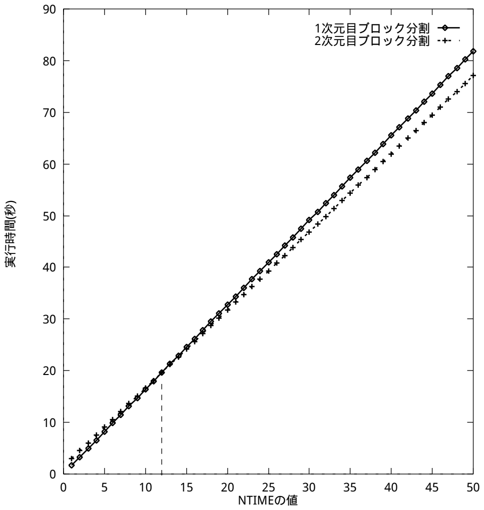
[8] 3^86GnFA: JBNs=hM}5;=Q, %3%m%J<R(1991).

IUO?!**livermore kernel No.23** $N<B9T;~4V$NHf3S

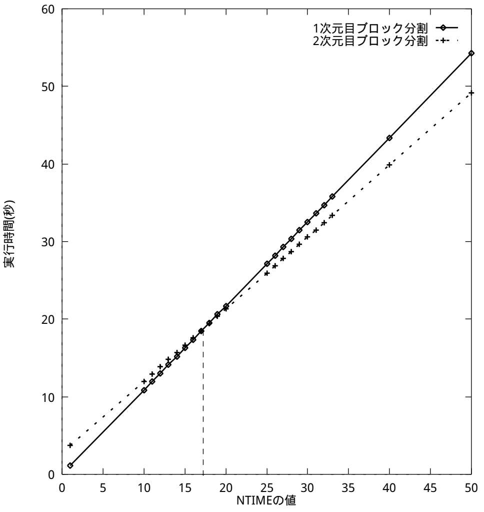
Bh5.2@a!@-G=8+@QKh$kJ,3d<!857hDj$NM-8z@-!W$G=R$Y$?livermore kernel No.23 KBP9$k<BB,$N<B9T;~4V$H!"@-G=8+@Q$jKh$k<B9T;~4V$rHf3S9k%0%i%U$r<($9!#3F%0%i%U$O!"1<!85L\%V%m%C%/J,3d$H2<!85L\%V%m%C%/J,3d$N3FJ,3d$KBP9kNTIME$H<B9T;~4V$N4X78$rI=$9!#3liO!"%W%m%;%C%5B f?t!"<BB,/8+@QNAH_9go;4H$K1$D$N%0%i%U$H7?!#



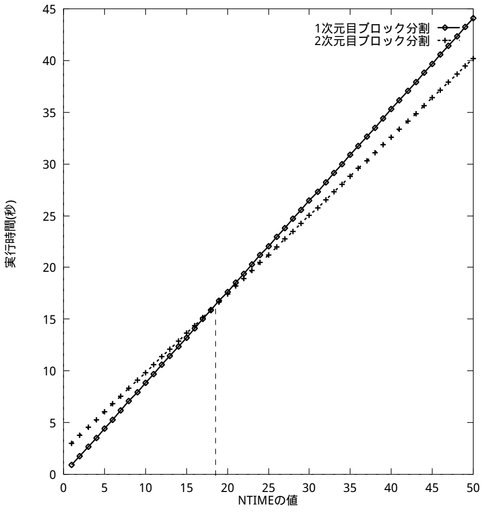
?^A.1: %W%m%;%C%5?t2(<BB,CM)



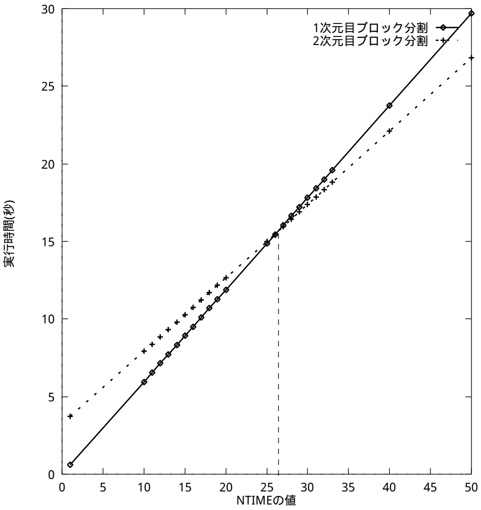
?^A.2: %W%m%;%C%5?t2(8+@QCM)



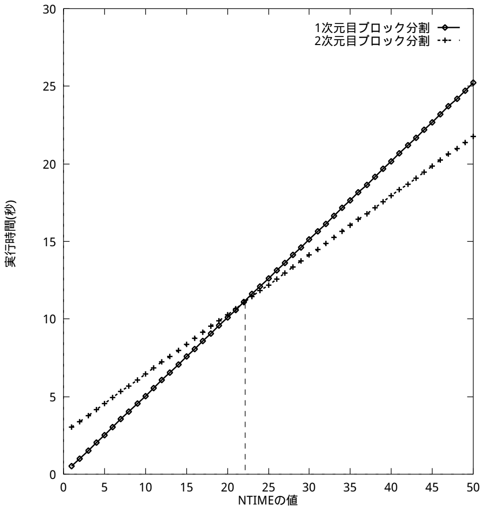
?^A.3: %W%m%;%C%5?t4(<BB,CM)



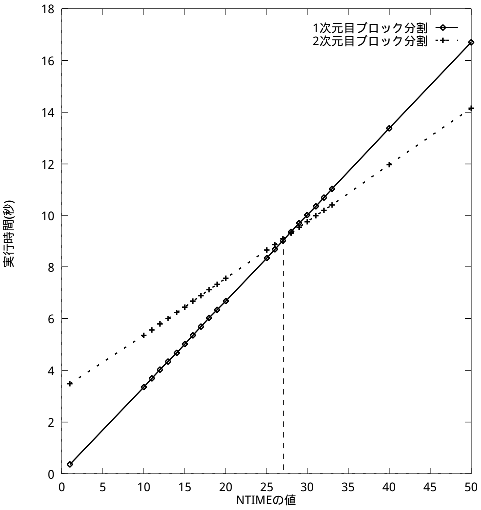
?^A.4: %W%m%;%C%5?t4(8+@QCM)



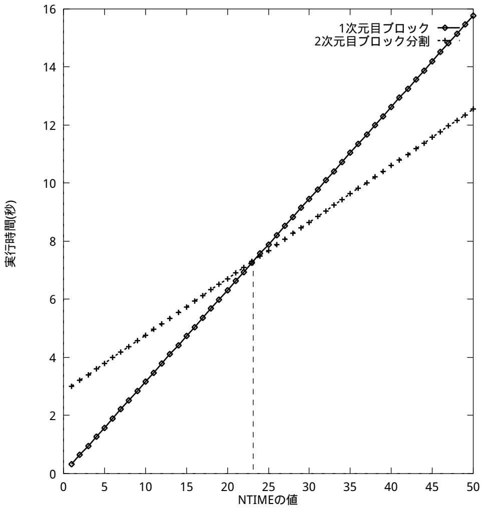
?^A.5: %W%m%;%C%5?t 8(<BB,CM)



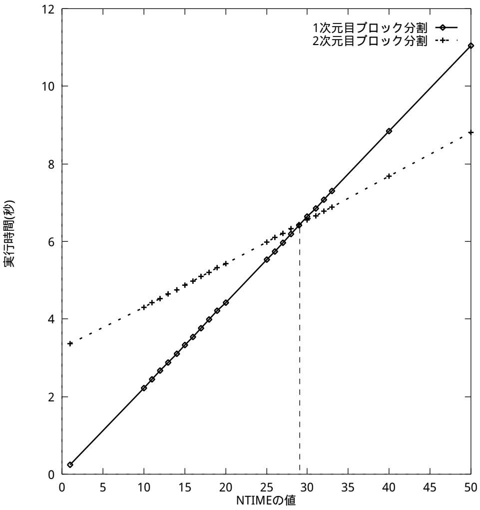
?^A.6: %W%m%;%C%5?t 8(8+@QCM)



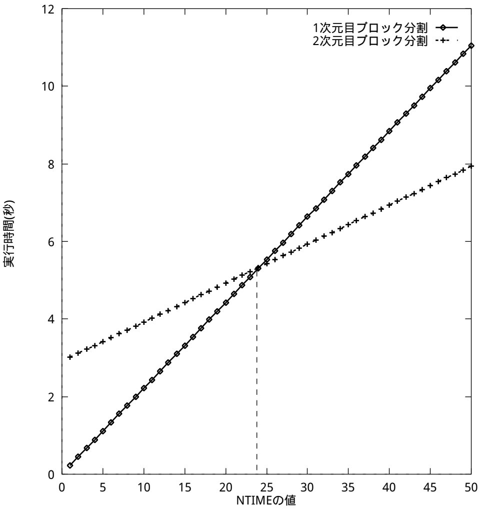
?^A.7: %W%m%;%C%5?t 16(<BB,CM)



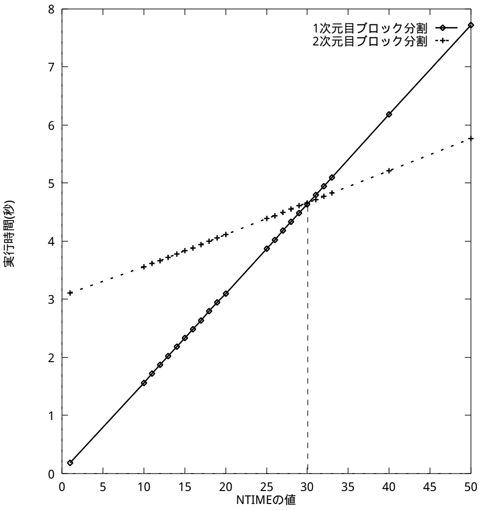
?^A.8: %W%m%;%C%5?t 16(8+@QCM)



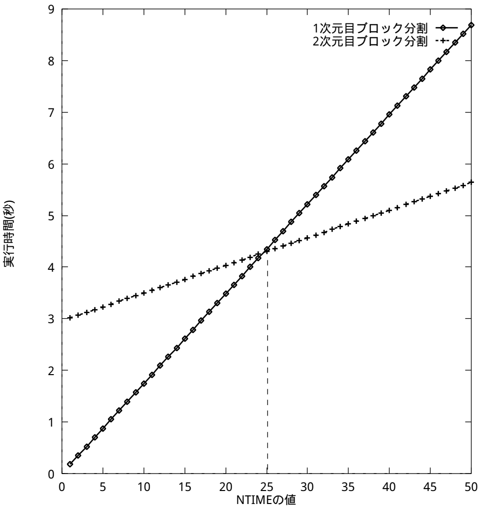
?^A.9: %W%m%;%C%5?t 32(<BB,CM)



?^A.10: %W%m%;%C%5?t 32(8+@QCM)



?^A.11: %W%m%;%C%5?t 64(<BB,CM)



?^A.12: %W%m%;%C%5?t 64(8+@QCM)